Estimating and Testing for Time-Varying stock market betas in Europe☆

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Abstract

This work analyzes the comovements between different domestic European stock market returns (Spain, France, Germany, Switzerland and United Kingdom) and the Eurostock and also the comovements between the Spanish index (IBEX) and the rest, not explained by the Eurostock. To do that, we estimate nonparametric betas to measure the possibly time-varying sensitivity of each country stock market to the European market returns. Accounting for the nonparametric specification, we propose a formal test for the constancy of these comovements, as a two-step procedure. A simulation study is presented to ensure the good performance. For the European data, the estimated betas show different patterns for different countries. On the one hand, when the test is applied to the relation between each domestic index and the Eurostock, all betas are significant and time-varying. On the other hand, we find that the IBEX has remaining time-varying comovements with the German and French index returns, not explained by the Eurostock, but not with the Swiss index. The relation with the UK index return disappears once the Eurostock effect is removed.

Keywords: Smoothing splines; Nonparametric estimation; Market model; Hypothesis testing.

1. Introduction

The analysis of the domestic stock markets and their relation with global indexes is interesting in many economic and financial aspects. Searching for market integrations, it is interesting to study the movements of relations between domestic indexes for European countries and an aggregate European index. Hardouvelis, Malliaropulos and Priestley (2006) study the integration degree with respect to an aggregate European index for twelve countries considered in their analysis. On the other hand, the most recent empirical evidence

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supports that the time-varying comovements between portfolios has become a risk, priced by the market in some contexts, as Driessen, Pascal and Vilkov (2009) or Krishnan, Petkova and Ritchken (2009) report. As a consequence, if the market integration is high, it would be expected than the risk due to time-varying comovements is mostly explained by the time-varying common factor loadings. That is, if the domestic market indexes are close to be integrated, their cross-loadings could be mostly explained by time-varying factor loadings. This is a very important fact, also from the macroeconomic point of view, due to the connection between stock market cross-country correlations and macroeconomic relations. Related to this, the recent work by Ülkü and Baker (2013) test the association between the stock market beta (the sensitivity of country stock market index to world index) and macroeconomic betas (the sensitivity of national output and inflation to world output and inflation) and Møller (2008) examines the ability of the consumption growth rate to capture predictable variation in stock returns over the business cycle.

Therefore, it becomes an interesting analysis to estimate the time-varying movements of the factor loadings for individual domestic market indexes. Specifically, we want to study how the Eurostock performs as a common factor for some European indexes. To do that, we propose to estimate the time-varying betas associated to several domestic indexes, where the market index is the Eurostock. We make the estimation for three countries belonging to the Eurozone (Spain, Germany and France) and two other out of the Eurozone (United Kingdom and Switzerland). It seems natural to think that the Eurostock performs as a market index better for the indexes of Eurozone countries. Within this framework, it is of interest to study the effectiveness of the European index as a common factor, for the Eurozone and non Eurozone countries. Another interesting issue is to test for the possibly remaining cross-movements between domestic indexes, once the Eurostock common influence is removed.

We propose to do both analysis under a time-varying pattern for the betas, which seems to be more suitable, according with the empirical evidence for time-varying betas in asset pricing models. In fact, in the three FF factors and the CAPM framework, it has become very common the use of nonparametric estimators to estimate this time-varying shape. See Ferson and Campbell (1999) for motivation and Esteban and Orbe-Mandaluniz (2010), Ferreira, Gil and Orbe (2011), Li and Yang (2011) and Ang and Kristensen (2012) and for different nonparametric estimation and testing proposals.

Here, we propose to use spline estimators to estimate the betas that capture the factor loadings between domestic market indexes and the global index. An corrected testing procedure is provided in order to test for the constancy of the betas, which takes into account the errors derived from the prior estimation of the unknown parameters. Without this correction, nonparametric testing procedures tend to over reject the null hypothesis, (see, e.g., Mammen (2000)).

Thus, the empirical objective is twofold. First, we estimate and test for the time-varying relation between the country indexes and the Eurostock index (EUROSTOXX 50). The data are daily stock indexes for Spain (IBEX 35), France (CAC 40), Germany (DAX), UK (FTSE 100) and Switzerland (SMI).

The time-varying estimation will allow to analyze the changes on the beta shapes and the nature of the comovements' changes for countries in and out of the Eurozone. Therefore, an interesting issue is to detect whether the time-varying correlation among pairs of indexes can be explained by the time-varying betas. Motivated by this, our second objective is to test for remaining comovementes between domestic indexes, once the relation with the Eurostock is removed. To do that, we will focus in the Spanish case. Thus, we test whether the IBEX has common comovements with other domestic indexes, once the comovements with the Eurostock are removed. Since the estimation is time-varying, the test is able to detect very flexible alternatives. We provide a procedure that avoid the distortion of the null hypothesis due to the error that comes from using nonparametrically estimated betas. To support the formal test, a simulation study is also presented.

The rest of the paper is structured as follows. Section 2 presents the methodology and Section 3 shows a simulation study. In Section 4 the methodology is applied to capture the time-varying comovements between five European indexes and the Eurostock and also to capture the remaining comovements between the Spanish index and the rest of the countries indexes. Section 5 concludes.

2. Methodology

Betas in the Market model.

The general parametric Market Model for one asset can be written as

$$R_{it} = \alpha_i + \beta_i R_{Mt} + u_{it} \quad t = 1, \ldots, T, \tag{1}$$

where R_M is the variable representing the global market risk, u is the error term and T is the sample size. For our interest, the asset i represents a domestic index.

As motivated in the introduction, we are interested in time-varying betas, so the model (1) is rewritten as

$$R_{it} = \alpha_i + \beta_{it} R_{Mt} + u_{it} \quad t = 1, \ldots, T. \tag{2}$$

The estimation of the model is done using smoothing splines techniques. The time-varying coefficients are estimated minimizing next optimization problem:

$$\min_{\alpha_i, \beta_{it}} \left(\sum_{t=1}^{T} (R_{it} - \alpha_i - \beta_{it} R_{Mt})^2 + \lambda_i \int (\beta_{it}'')^2 \, dt \right)$$

where the first part measures the goodness of fit to the data and the second part accounts for the smoothness of the curve of betas.

An important issue for a proper application of this nonparametric estimation method is to provide an automatic selection of the smoothing parameter λ_i, which decides the best trade-off between the two parts (see Wahba (1990)). We use cross-validation for this selection, using the algorithm provided in the function `gam(.)` of the package `mgcv` (Wood (2004)) for the statistical software environment R of public domain. The amount of smoothing is expressed by the "Estimated Degrees of Freedom" (EDF), which can be seen as a continuous version of the number of parameters used for fitting the curve since the smoothing matrix plays a similar role to the hat matrix in linear regression. Therefore, by taking the trace of the smoothing matrix we obtain the EDF, that indicates the complexity of the underlying function and that is used to calculate the testing statistics.

Eisenbeiss, Kauermann and Semmler (2007) use this nonparametric estimation methodology to estimate and test time-varying betas for the German stock market. They split β_{it} into two parts, a constant part plus a time-varying part, $\beta_i + \widetilde{\beta}_{it}$, introduce it into the model and estimate the resulting additive model by the smoothing spline technique described above using the package written by Wood. The output of this estimation offers among others results, the estimated coefficients $(\widetilde{\alpha}_i, \widetilde{\beta}_i, \widetilde{\beta}_{it})$, measures of goodness, EDFs, Wald-type test-statistic values for testing significance (adjusted by EDFs) of all terms as well as their associated p-values.

However, although the time-varying coefficients $(\beta_i + \widetilde{\beta}_{it})$ are estimated adequately, there is an identification problem estimating independently the two parts and consequently, they cannot be used to test neither significance or constancy. To avoid this problem in testing, we propose a two-step testing procedure. First, we estimate the constant part using OLS and second, we use the residuals to estimate the time-varying part using GAM. As mentioned in the introduction, the use of the same sample for estimating and testing can lead to distortions of test size so an adequate subsample must be taken for each case.

In practice, the procedure to test for time-varying betas goes as follows:

(i) For each series of returns R_i, estimate α_i and β_i using OLS in the constant coefficient model (1).

(ii) Take a uniformly spaced subsample of the residuals ($\hat{u}_{is} = R_{is} - \widehat{\alpha}_i - \widehat{\beta}_i R_{Ms}$), of size $T_S = o(T)$.

(iii) Estimate the time-varying part of the coefficients, $\widetilde{\beta}_{it}$, considering next model

$$\hat{u}_{is} = \widetilde{\beta}_{is} R_{Ms} + v_s \quad s = 1, \ldots, T_S. \tag{3}$$

where the explained variable are the residuals of the constant coefficient model estimated by OLS and the error v_s is considered i.i.d.

(iv) Compute the associated Wald-type F-statistic and reject the null of constant coefficients, $\widetilde{\beta}_{is} = 0 \, \forall s$, if the F-statistic is large enough or equivalently, if the p-value is small. The rejection implies a time-varying-coefficient model whereas the no rejection indicates that the OLS estimation of a constant coefficient model is adequate for the data.

Covariance between domestic indexes.

Consider now two different countries and write model (1) for the respectively domestic indexes

$$R_{it} = \alpha_i + \beta_i R_{Mt} + u_{it} \tag{4}$$

$$R_{jt} = \alpha_j + \beta_j R_{Mt} + u_{jt} \tag{5}$$

The time-varying specification allows us to make a more general interpretation of the comovements. Under this framework, if the linear comovements between two indexes are only due to the common risk factor, we should find that the covariances between both error terms is zero; that is, $Cov(u_{it}, u_{jt}) = E(u_{it}u_{jt}) = 0 \quad \forall i \neq j$. Thus, zero covariance implies that the correlation between the two domestic stock markets is only due to the correlation between each of them and the Eurostock. On the contrary, if the covariance is not zero, we have that there exists some comovements that are not explained only by the Eurostock.

Moreover, if the commovements between two indexes are time-varying and the covariance between the error terms is constant, we can state that the time-varying structure of the comovements is due to the time-varying influence of the Eurostock and viceversa. This issue is important not only to identify the source of the time-varying behavior in commovements, but also to find which and how many are the risk factors in the comovements structure between countries.

Next we provide the methodology to estimate betas and test for nonzero intercept and covariances, considering that the stock market betas are nonparametrically estimated. To test for zero (constant) covariance between two series R_i and R_j when the influence of the market is removed, the procedure is:

(i) Estimate a time-varying coefficients model nonparametrically for each country and take the residuals: $\hat{u}_{it} = R_{it} - \widehat{\alpha}_i - \widehat{\beta}_{it} R_{Mt}$.

(ii) Use a subsample of order $T^{4/5}$ to estimate nonparametrically the coefficients in next auxiliar regression model $\hat{u}_{is} = \sum_{j \neq i} \gamma_{i,j,s} \hat{u}_{js} + e_i$.

(iii) Compute the respective F-statistics to test for significance or constant coefficients and conclude.

The same nonparametric methodology and the same F-statistic as before applies. In summary, for a given country i, we can test for the following situations

1. Significance of the beta associated with the global index.
2. Time-varying performance of the beta.
3. Significance of the comovement with another domestic index, once the effect of the global index is removed.
4. Time-varying performance of the comovement.

Before applying the methodology to real data of interest, we provide results in a simulation study to guarantee the good performance in practice.

3. A simulation study

This estimation and testing procedure is robust against general alternatives. In fact, the test can be applied against alternatives without the need of any parametric specification. To support the use of this procedure the following simulation study is made.

Define the general data generating process as

$$Y_t = 1 + \beta_t X_t + u_t \quad t = 1, \ldots, T,$$

with

$$\beta_t = 2 + b\left(\frac{t}{T} - 0.5\right)^2 + c\,\sin\left(6\pi\frac{t}{T}\right),$$

and where the fixed parameters b and c allow for different shapes in beta. Taking different values those parameters, we apply the procedure and compute the percentage of simulations in which the null of a constant beta is rejected. For $b = c = 0$ the size is computed and for the rest of cases the power is obtained.

Motivated by the empirical evidence in financial data we consider, for each set of parameters, different X_t and u_t. For X_t we consider realizations from a uniform and from a GARCH distribution. The errors u_t are simulated from a *i.i.d.* Normal and from a GARCH distribution.

The data generating process for X_t are $X_t \sim U(3, 15)$ and a GARCH(1,1) such that $X_t = 9 + h_t\varepsilon_t$, where ε_t are *i.i.d.* $N(0, 1)$ and $h_t^2 = 3.4 + 0.2\varepsilon_{t-1}^2 + 0.7h_{t-1}^2$. The realization is fixed for each case in the simulation study, to control the size of the test. For the errors, two alternatives are also considered, $NI\left(0, \sigma^2 = 5\right)$ and $u_t = g_t\epsilon_t$ with ϵ_t *i.i.d.* $N(0, 1)$ and $g_t^2 = 2.3 + 0.2\epsilon_{t-1}^2 + 0.5g_{t-1}^2$.

Size

We have performed the beta estimation using three procedures. First, we estimate both parts at the same part using the GAM method and test for the significance of the time-varying part without correcting the statistic. As expected, the identification problem appears and the method is unable to split the beta into two terms such that the time-varying part moves around zero. Hence, the null hypothesis is rejected in all cases and the percentage of rejections is almost one in all cases (detailed tables are therefore omitted). Second, the two-step procedure is used but with the same sample size (without subsampling) to estimate and test, the results show an over rejection of the null. Finally, the two-step procedure using the subsampling leads to very accurate sizes, for all different combinations. This results are presented in Table 1 for different sample sizes. As it can be observed this testing procedure provides good results even with small sample sizes.

Power

Table **??** present the rates of empirical rejections under different alternatives. As expected, as long as the sample size increases and also when the alternative is further from the null, the power increases. It is remarkable that even for moderate sample sizes, the subsampling method has a good performance even for parameters close to the null hypothesis.

Table 1: Empirical size.

	With subsampling			Without subsampling		
X Uniform, u *iid* Normal	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$
$T=250$	.0074	.0454	.0976	.0042	.0278	.0568
$T=500$	.0082	.0506	.1068	.0046	.0228	.0464
$T=1000$	.0068	.0386	.0928	.004	.022	.0476

X GARCH, u *iid* Normal	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$
$T=250$	.0096	.0492	.1018	.0048	.0244	.0538
$T=500$	.0088	.0454	.0952	.0022	.0176	.0456
$T=1000$	.0088	.0484	.0986	.0038	.0214	.0486

X GARCH, u GARCH	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$
$T=250$	.009	.0456	.0932	.0048	.0222	.0508
$T=500$	.0086	.0438	.0958	.0044	.0238	.05
$T=1000$	.0058	.0424	.0924	.0018	.0194	.048

X GARCH, u GARCH	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$	$\alpha=.01$	$\alpha=.05$	$\alpha=.1$
$T=250$	.011	.0476	.099	0.0048	0.0238	0.0498
$T=500$	.0084	.0504	.095	0.004	0.0258	0.0518
$T=1000$	.0056	.0408	.0982	0.003	0.0202	0.0456

This table presents the empirical rate of rejections under the null of constant betas for four cases and three different sample sizes: 250, 500 and 1000. The cases are the combinations of two different specifications for X (Uniform and GARCH) and u (*iid* Normal and GARCH). X Uniform (GARCH) means that X is a realization of a uniform distribution $U(3,15)$ ($X_t = 9 + h_t\varepsilon_t$ with ε_t *iid* standard normal distributed and $h_t^2 = 3.4 + .2\varepsilon_{t-1}^2 + .7h_{t-1}^2$). u *iid* Normal is drawn from a $NI(0, \sigma^2 = 5)$ whereas the GARCH specification is modelled as $u_t = g_t\epsilon_t$, with ϵ_t *iid* standard normal distributed and $g_t^2 = 2.3 + .2\epsilon_{t-1}^2 + .5g_{t-1}^2$.

4. Application to the European Stock Market.

In this section, we apply the methodology to analyze the relation between the Eurostock and several European domestic indexes as well as the relation between the Spanish index, IBEX35 and the other domestic indexes, once the effect of the Eurostock is removed.

We have considered daily indexes[1] for five different countries: Spain (SP),

[1] Source: http://finance.yahoo.com/.

Germany (GR), France (FR), Switzerland (SW) and United Kingdom (UK). Spain, Germany an France belong to the Eurozone and they have a strong representation in the Eurostock index. France has a relative weight of 36%, Germany 32% and Spain 12%. On the other hand, United Kingdom and Swiss are selected to illustrate the performance of two important European western countries, non-euro countries. The sample runs from 1994-03-01 to 2103-11-28 and contains the closing daily data for each index.

Figure 1 presents the time-varying evolution of the four indexes returns. The curves are obtained smoothing the returns with a smoothing spline. Looking at the figure, we see a time-varying evolution with common patterns between them. The first question is how much these time-varying evolutions are explained by the Eurostock. The second is whether or not the Eurostock is able to explain partially or totally the common evolutions.

FIGURE 1 ABOUT HERE

4.1. Domestic betas.

We measure and test the relation between the domestic indexes and the global Eurostock index. First, we estimate the time-varying betas following the methodology described in Section 2. The model is

$$R_{it} = \alpha_{it} + \beta_{it} R_{EUt} + u_{it} \quad t = 1994 - 03 - 01, \ldots, 2103 - 11 - 28. \tag{6}$$

where R_{EU} denote the excess returns for the Eurostock index and R_i represents the excess return for each domestic index $i = SP, GR, FR, SW, UK$. For homogeneity, the excess returns are obtained using one-month US Treasury bill rate as the proxy for the risk-free interest rate. Table 3 presents the main descriptive statistics for all excess returns considered.

Table 3: Eurostock and domestic indexes returns. Summarized statistics

Index	Minimum	Maximum	Mean	SD	Skewness	Kurtosis
EUROSTOXX	-0.078853	0.138246	0.000177	0.015345	0.201228	5.484541
IBEX 35 (SP)	-0.108617	0.144253	0.000237	0.015902	0.188885	5.921438
CAC 40 (FR)	-0.090417	0.142291	0.000150	0.015480	0.251012	5.488136
DAX (GR)	-0.091506	0.144053	0.000334	0.015975	0.146045	5.096494
FTSE 100 (UK)	-0.088502	0.117480	0.000116	0.012499	0.152602	6.969483
SIM (SW)	-0.077902	0.170704	0.000196	0.012819	0.496736	11.26624

This table presents the main descriptive statistics for the six analyzed indexes. The data are daily excess returns for the Eurostock and the five countries considered: Spain (SP), Germany (GR), France (FR), United Kingdom (UK) and Switzerland (SW). The risk-free interest rate is the one-month T-Bill.

As can be observed, for all countries the intercepts are not significant whereas the betas are all significant and time-varying. This implies that the expected excess return of each domestic index minus the expected excess return of the Eurostock times the time-varying beta has zero expectation. Moreover, the results support a time-varying performance of the betas, although in terms of adjustment improvement, either measured by the R^2-adj or by the GCV, the aggregate results do not vary substantially. That is, the differences between constant and time-varying betas are not relevant in terms of a explaining the variability of the expected returns.

Figure 2 presents the estimated time-varying betas together with the 95% confidence intervals. For each country the figure shows the movements of the respective domestic betas. Germany and France present the most stable pattern, whereas Swiss and UK betas present more oscilations. In average, the betas are

Table 4: Inference for domestic betas.

$Index$	$Constant$				$Time-varying$			
	p-value ($\alpha_i = 0$)	p-value ($\beta_i = 0$)	R^2-adj.	GCV	p-value ($\tilde{\alpha}_{it} = 0$)	p-value ($\tilde{\beta}_{it} = 0$)	R^2-adj.	GCV
IBEX 35 (SP)	0.504	<2.2e-16	0.771	5.80e-05	0.568	3.44e-09	0.779	5.59e-05
CAC 40 (FR)	0.776	<2.2e-16	0.909	2.19e-05	0.865	5.39e-06	0.910	2.15e-05
DAX (GR)	0.080	<2.2e-16	0.839	4.10e-05	0.562	4.59e-05	0.843	4.02e-05
FTSE 100 (UK)	0.928	3.96 e-05	0.747	<2.2e-16	0.228	2.27e-12	0.763	3.70e-05
SIM (SW)	0.491	<2.2e-16	0.683	5.21e-05	0.147	<2.2e-16	0.704	4.88e-05

This table presents the inference results for the different estimates. The first vertical block defined as $Constant$ shows the OLS results for a constant coefficient model: $R_{it} = \alpha_i + \beta_i R_{EUt} + \epsilon_{it}$. In this blocks there is the p-values for the test of significative intercept and significative betas, together with the R^2-adj and the Generalized Cross Validation value (GCV) of the model. The second block, called $Time-varying$, shows the testing results of a time-varying coefficient model: $R_{it} = \alpha_{it} + \beta_{it} R_{EUt} + \epsilon_{it}$. In this block the p-values are for the test of constant intercepts and constant betas, together with the R^2-adj and the Generalized Cross Validation value (GCV).

larger for the Eurozone countries than for the rest, which seems a natural result.

FIGURE 2 ABOUT HERE

In summary, the Eurostock appears to be explain a high proportion of the expected excess returns for the domestic indexes. Moreover, the beta parameters are time-varying for all domestic indexes. Therefore, this common effect could explain the relation between pairs of domestic indexes. To test for this, we analyze the remaining relations between the residuals when the Eurostock effect is removed. The case of Spain will be studied in the following section.

4.2. The case of Spain.

Using this methodology, we can now estimate the remaining linear relation between each country and the rest, when the effect of Eurostock is removed. That is, we are interesting in the linear relation between u_{it} and the rest of u_{jt}.

In particular, we consider the case of Spain, and the model becomes

$$\begin{aligned} u_{SPt} &= \gamma_{SP,t} + \gamma_{SP,GR,t}\, u_{GRt} + \gamma_{SP,FR,t}\, u_{FRt} \\ &\quad + \gamma_{UK,GR,t}\, u_{UKt} + \gamma_{SP,SW,t}\, u_{SWt} + \varepsilon_{SPt}. \end{aligned} \tag{7}$$

If there are no remaining comovements, and the betas from the new model above are zero, the relation between the Spanish index (IBEX) and the rest of the countries indexes is totally explained by the Eurostock. In other case, a nonzero beta for a index return j indicates that the expectation of the error term for the IBEX conditional to the index return j is different from zero and, therefore, some information can be captured from this relation.

To estimate this relation, we define $\hat{u}_{it}$ as the residuals from (6): that is, $\hat{u}_{it} = R_{it} - \hat{\alpha}_{it} - \hat{\beta}_{it} R_{EUt}$ and we replace the variables in model (8) by their estimates $\hat{u}_{it}$. Therefore, the final betas are estimated from model(8).

$$\begin{aligned} \hat{u}_{SPt} &= \gamma_{SP,t} + \gamma_{SP,FR,t}\, \hat{u}_{FRt} + \gamma_{SP,GR,t}\, \hat{u}_{GRt} \\ &\quad + \gamma_{UK,GR,t}\, \hat{u}_{UKt} + \gamma_{SP,SW,t}\, \hat{u}_{SWt} + \varepsilon_{SPt} \end{aligned} \tag{8}$$

The OLS estimator provides the result

$$\begin{aligned} \hat{u}_{SPt} &= 6.113 \times 10^{-05} - 9.009 \times 10^{-02}\, \hat{u}_{FRt} - 2.628 \times 10^{-01}\, \hat{u}_{GRt} \\ &- 1.631 \times 10^{-02}\, \hat{u}_{UKt} + 7.113 \times 10^{-02}\, \hat{u}_{SWt} \end{aligned} \tag{9}$$

It is interesting to compare the results with those obtained when the model (8) is estimated under constant betas. The results are presented in Table 5. For the ease of interpretation and comparison we present separately the p-values corresponding to the OLS and to the time-varying estimation parts. For the OLS, we show the p-values used to test for significance of the constant terms. For the time-varying part, we present the p-values for the significance of the time-varying parts, once the OLS estimation is removed. As proposed in Section 2 for testing purpose, we consider a subsample with the appropriate size for the nonparametric case, to avoid distortion of the null due to beta estimation errors.

The results are very interesting. We find that there is no extra relation between the IBEX returns and the index returns in UK; that is, the relation between these indexes seems to be totally explained by the Eurostock. When the relation with the Swiss index is analyzed, some relation is still remaining, but not time-varying. Surprisingly, the domestic indexes from the Eurozone, DAX and CAC still share comovements with the Ibex, significant and time-varying. Moreover, the adjusted R^2 takes the value 0.16 when the time-varying model is estimated, which increases almost a 50% the R^2 of the constant model. That is, the improvement is significant and the difference between the adjustment seems to be relevant.

To illustrate these relations, Figure 3 presents the estimated time-varying comovements between the residuals, together with the 95% confidence interval. It seems very clear the remaning relation with the German and the French indexes. It is also interesting to point out that, once the Eurostock has been removed, the relation between the Spanish index and the rest has become decreasing and negative in the last years. This coincides with a high effect of the crisis in Spain.

FIGURE 3 ABOUT HERE

5. Main conclusions

A nonparametric methodology suitable for estimating and testing for time-varying betas has been proposed. The procedure is very easy to implement and robust to estimation errors. It uses splines for estimation and a subsampling procedure for the testing part. The performance is analyzed in a simulation study, which shows a good performance of the testing procedure. The proposed methodology is used to estimate the factor loadings between domestic market indexes and the Eurostock. The results show that these factor loadings are time-varying, although the improvement due to the use of varying betas is not substantially very large, in terms of global measures of accuracy.

Table 5: Inference in the covariance model.

j	p-value ($\gamma_{SP,j}=0$)	p-value ($\gamma_{SP,j,t}=0$)	p-value ($\tilde{\gamma}_{SP,j,t}=0$)
Intercept	.7788	.558	.558
CAC 40 (FR)	.0732	2.54e-08	1.05e-07
DAX (GR)	2.73e-14	$<$2e-16	$<$ 2e-16
FTSE 100 (UK)	.6685	.6792	.487
SIM (SW)	.0347	.0036	.116

This table presents the p-values of testing significance and constancy for the covariance between domestic indexes using model (8). The first column shows the p-values of testing significance assuming constant coefficients and using the OLS estimator. Second and third columns show p-values for testing significance and constancy of the coefficients associated to the j-th serie of residuals.

The methodology is also applied to study the relation between the Spanish index and the rest of domestic indexes, when the effect of Eurostock is removed. For this case, the results show that there are remaining comovements with Germany and France, not explained by the Eurostock. However, the time-varying comovements with the Swiss and the UK indexes are only due to the respectively comovements with the global index.

The methodology is very flexible and do not assume any parametric pattern for the comovements so the results are robust for nonparametric beta specifications. This procedure can be applied to any beta pricing model, to find which and how many risk factors are needed to explain comovements between the assets or portfolios considered in the model.

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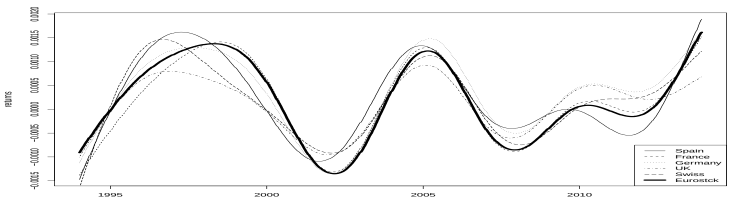
Figure 1: Return's smoothed trends



This figure shows the smoothed path of the four indexes returns along time.

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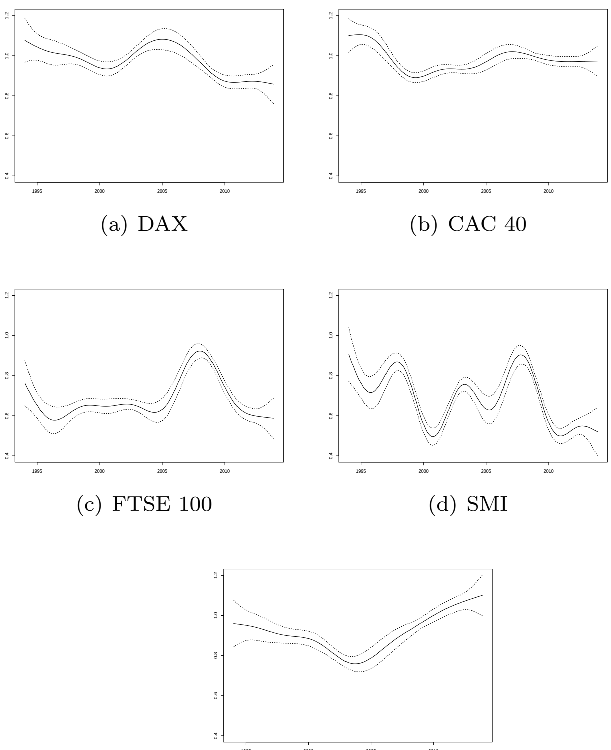
Figure 2: Estimated time-varying domestic betas.



(a) DAX

(b) CAC 40

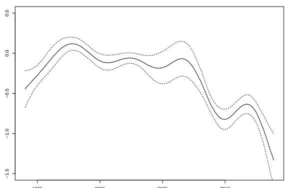
(c) FTSE 100

(d) SMI

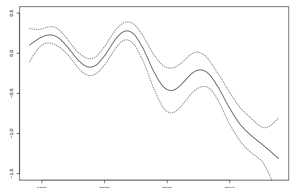
(e) IBEX 35

This figure shows the estimated time-varying beta with its 95% confidence interval for each country. The estimated regressions are $R_{it} = \alpha_j + \beta_{it} R_{Mt} + u_{it}$ $\quad t$=1994:01:03,..., 2013:11:28 $\quad i$ =FR, GR, SW, UK, SP.

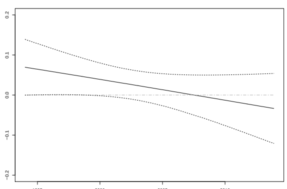
Figure 3: Estimated time-varying coefficients for each covariable.



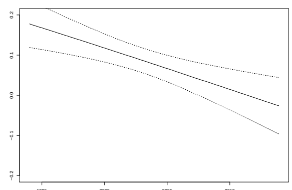
(a) DAX



(b) CAC 40



(c) FTSE 100



(d) SMI

This figure shows the estimated time-varying beta with its 95% confidence interval for each explanatory variables. The model estimated is $\hat{u}_{SPt} = \gamma_{SP,t} + \gamma_{SP,FR,t}\,\hat{u}_{FRt} + \gamma_{SP,GR,t}\,\hat{u}_{GRt} + \gamma_{UK,GR,t}\,\hat{u}_{UKt} + \gamma_{SP,SW,t}\,\hat{u}_{SWt} + \varepsilon_{SPt} \quad t = 1994:01:03, \ldots, 2013:11:28.$