INVESTING IN THE EUROPEAN SIZE FACTOR

Juan Laborda [a], Ricardo Laborda [b,*], Jose Olmo [c]

[a] Instituto de Estudios Bursátiles. Calle Alfonso XI 28014, Madrid, Spain
[b] Centro Universitario de la Defensa, Academia General Militar. Ctra Huesca s/n 50090, Zaragoza, Spain
[c] Department of Economics, University of Southampton, United Kingdom

Abstract

This paper investigates the role of the size factor for constructing investment portfolios and proposes a dynamic extension that accommodates the risk-free asset and time-varying weights. These weights are determined by a set of state variables given by the term structure of sovereign interest rates, variables describing market risk aversion such as the VIX index and the CRB Industrial return, and indexes reflecting investor sentiment towards the economic outlook. The empirical analysis explores the performance of a naïve European size factor and compares it against the dynamic version that optimizes the weights in each period. The results provide support for the European size factor except for periods of economic distress in which the optimal dynamic strategy is clearly superior.

Key Words: European size factor, GMM methods, Optimal parametric portfolio, Out-of-sample test, State variables

JEL Classification: G11, G15, F31.

*Corresponding author. Tel.: +34 9767076. e-mail address: rlaborda@unizar.es .

1. Introduction

Fama and French (1993, 1996) show that the empirical power of the standard Capital Asset Pricing Model for explaining the cross-sectional variation in average equity returns can be highly improved if the portfolios are sorted by size and book-to-market ratios. The components of this three-factor model are specifically defined as the excess return on a market portfolio (Mkt factor), the return on a portfolio that holds long positions in small capitalization stocks (small caps) and short positions in large capitalization stocks (SMB factor), and the return on a portfolio that holds a long position in high book-to-price stocks and a short position in low book-to-price stocks (HML factor).

The financial literature has focused on the search of the economic sources that could elucidate rationally the ability of these factors to explain the cross-sectional variation in average returns of portfolios, giving a risk-based explanation behind the SMB and HML factors. The interest in exploring the profitability of portfolios replicating these factors resides in their ability to capture future changes in the economic environment, such as the impoverishment in credit conditions reflected by an increase of the business failure rates, see Kapadia (2011), changes in the level of economic activity, see Liew and Vassalou (2000), or changes in financial market conditions, such as countercyclical dynamics of the Sharpe ratio, see Quiros and Timmermann (2001). It is, therefore, possible to establish a relationship between the investment strategies replicating the SMB and HML factors and macroeconomic and financial risks. Thus, one of the main motivations of this paper is to find business cycle related variables usually linked to market risk premiums that anticipate the dynamics of these factors.

In this paper we focus on the SMB factor with data from European financial markets extending the evidence on the importance of the size factor for U.S. financial markets. Our choice of the SMB factor as opposed to the HML factor is due to its higher correlation with variables proxying credit market conditions, monetary policy and business cycle dynamics. More specifically, our aim is to investigate the performance of a portfolio that replicates the returns on the SMB factor and explore different sets of macroeconomic variables with power to predict its dynamics. This is particularly relevant for envisaging dynamic investment strategies providing hedging against downturns of the SMB factor and correlated with macroeconomic conditions. The dynamic hedging strategy is achieved by making allowance for a risk-free asset that completes the investment opportunity set available to the investor. In this simple investment framework the optimal strategy consists of a vector of time-varying weights that allows investors with interests in positioning on the SMB factor to switch between long and short positions in the risk-free asset and the SMB factor depending on the extant macroeconomic conditions in each period. Investors obtain their optimal strategy by maximizing in each period the expected utility on their portfolio return under the assumption that the returns on the SMB factor are linearly related to a set of macroeconomic variables proxying its dynamics. This methodology has been originally exploited in optimal portfolio theory in the seminal articles by Brandt (1999), Ait-Sahalia and Brandt (2001) or Brandt, Santa Clara, and Valkanov (2009).

In our empirical analysis, we observe that the set of macroeconomic variables with power to explain the SMB factor is related to the yield curve on European sovereign debt, the overall credit conditions, market risk aversion and investor economic sentiment. The performance of the dynamic optimal portfolio is compared against the passive SMB portfolio strategy by means of statistical and economic measures such as the Sharpe and Sortino ratios and the difference in certainty equivalent returns (CERs) between strategies obtained under the assumption that individual preferences exhibit constant relative risk aversion. The main findings of the empirical section are summarized as follows.

First, we provide empirical support for the existence of an investment style characterized by a portfolio tracking the returns on a European size factor. The return on this portfolio is determined by the differences in performance between small and large European firms in terms of asset capitalization. The portfolio corresponding to this size factor investment style performs very well during economic expansions but poorly over recession episodes and periods of high uncertainty in financial markets. Large firms outperform small firms during these turmoil periods. From an empirical perspective these findings motivate the construction of simple dynamic strategies allowing for short positions in the size factor with the aim of hedging against economic downturns. Thus, our results over a sample covering twenty years of daily returns clearly show the statistical and economic gains from switching between short and long positions in the European size factor. These results are confirmed out-of-sample and especially after the subprime crisis, for which the tactical asset allocation problem suggests that following the passive strategy but avoiding episodes of increasing market risk aversion delivers a high reward to the investor. Second, our in-sample and out-of-sample analyses of the dynamics of the optimal portfolio reveal that the weight function determining the investor's optimal position on the European size factor is negatively related to the level factor of the German sovereign yield curve, the level factor of the German sovereign yield curve momentum, the German economic sentiment index and the three month change in the VIX index. It is also positively related to the three month change of the CRB Industrial return. The importance of these variables in determining the optimal portfolio allocation resides in its ability in predicting the SMB factor over our evaluation periods. Thus, the empirical relationship between interest rates and the size factor can be rationalized by noting that low and decreasing interest rates can be associated to easier access to private financing by small caps, enhancing, in turn, their relative valuations compared to large cap stocks. A low interest rate environment also favors the relative performance of small caps compared to large caps by reducing the existence of agency costs in credit markets borne by the former group. The negative relationship between investor sentiment and the size factor can be explained by observing that periods of tightening in monetary policy are usually associated to potential overvaluations of small caps stocks leading to price corrections and yielding negative returns. The relationship between the VIX index and the CRB Industrial return with the size factor is through market expectations on increased risk premiums and the deterioration of the economic outlook. More specifically, positive variations of the VIX index and negative of the CRB Industrial return signal increased risk aversion levels and a drop in commodity prices, yielding in turn, a higher probability of shorting the European size factor.

The rest of the article is structured as follows. Section 2 presents the methodological background needed to construct the optimal dynamic investment strategies. Section 3 discusses the choice of state variables proxying the dynamics of the SMB factor. The empirical application in Section 4 illustrates the differences in performance between the passive SMB strategy and its dynamic counterpart considering the state variables in the optimization problem. Section 5 concludes. Tables and figures are collected in an appendix.

2. Methodological Background

The methodology to derive the dynamic optimal portfolio strategy combines two strands of the literature on optimal portfolio allocation. First, we build on the recent literature by Brandt (1999), Ait-Sahalia and Brandt (2001) and Brandt et al. (2009) and propose portfolios with optimal weights determined by linear functions of some state variables with macroeconomic content. This approach establishes a link between the distribution of the returns on the investor's portfolio, the macroeconomic environment and investor's preferences. Second, we explore the empirical literature assessing the relationship between the economic cycle, market risk aversion and economic sentiment and the relative performance of small and large firms. The existence of empirical correlations between these variables underpins our choice of state variables proxying the set of information available to the investor and relevant for determining their dynamic investment strategies.

2.1 Small caps and large caps: an asset allocation approach

This section presents the problem of an investor who needs to allocate their wealth between a risk-free asset with return rf_t and a risky portfolio, denominated hereafter as the naïve size factor portfolio, with return $R_{SMB,t+1}$. The latter portfolio replicates the SMB factor and is defined in our paper as the difference between the Eurostoxx 50 and the Stoxx Europe Small 200. The optimal portfolio weight $\alpha_{SMB,t}$ allocated to the size factor is defined as a linear function of all the state variables that are expected to have predictive power for describing the dynamics of this factor and potentially affect investor's expected marginal utility, see for example Brandt (1999). These variables are related by the following portfolio return specification:

$$R_{p,t+1} = \alpha_{rf,t} rf_t^{EU} + \alpha_{SMB,t} R_{SMB,t+1},$$

with rf_t^{EU} the return on an European risk-free asset, $R_{t+1,SMB}$ the return on the European size factor, and $\alpha_{rf,t}$ and $\alpha_{SMB,t}$ the share of investment in each asset and such that $\alpha_{rf,t} + \alpha_{SMB,t} = 1$. Hence, we assume that the investor can invest directly in a security that replicates the SMB strategy return. This portfolio can be expressed as

$$R_{p,t+1} = rf_t^{EU} + \alpha_{SMB,t} R_{SMB,t+1}^{e}, \tag{1}$$

with $R_{SMB,t+1}^{e}$ the excess return on the size factor. We impose the assumption $-1 < \alpha_{SMB,t} < 2$ that avoids excessive and unrealistic leverage. Values of $\alpha_{SMB,t}$ between -1 and 0 reflect short positions on the size factor and a long position on the risk-free asset; values between 0 and 1 reflect a long position in both assets in the portfolio; a value of $\alpha_{SMB,t}$ between 1 and 2 signals a short position on the risk-free asset compensated by a significant long position on the size factor. The case $\alpha_{SMB,t} = 1$ corresponds to the passive strategy given by replicating the SMB factor during that period. The optimal portfolio weight $\alpha_{SMB,t}$ is defined as

$$\alpha_{SMB,t} = \alpha\left(Z_t;\beta\right) = Z_t^{'}\beta, \tag{2}$$

with beta a vector of coefficients to be optimally selected. The investor's optimal asset allocation problem is to maximize its expected utility conditional on the sigma-algebra determined by the available information set. This problem is mathematically stated as

$$\underset{\beta}{Max}\, E\left[U\left(R_{p,t+1}(\alpha(Z_t;\beta))\right)\middle|\Omega_t\right], \tag{3}$$

with $U(\mathrm{R}_{p,t+1};\beta)$ denoting investor's utility, Ω_t the corresponding sigma-algebra and $E\left[\bullet\middle|\Omega_t\right]$ the mathematical expectation conditional on Ω_t. The first-order conditions of this maximization problem are

$$E\left[U'\left(R_{p,t+1}(\alpha(Z_t;\beta))\right) R^e_{SMB,t+1}\middle|\Omega_t\right]=0, \tag{4}$$

with $U^{'}\left(R_{p,t+1};\beta\right)$ denoting investor's marginal utility. Further, under the assumption that the information contained in Ω_t is completely reflected by the state vector Z_t, the above condition implies that

$$E\left[U^{'}\left(R_{p,t+1}(\alpha(Z_t;\beta))\right) R^e_{SMB,t+1} \otimes Z_t\right]=0, \tag{5}$$

with $\otimes$ denoting element-by-element multiplication.

2.2 Estimation of the model parameters

The above representation of the optimal asset allocation problem yields a testable representation that can be implemented using generalized method of moments (GMM) techniques. Let $h\left(R_{p,t+1},Z_t;\beta\right)=U^{'}\left(R_{p,t+1};\beta\right) R^e_{SMB,t+1}\otimes Z_t$ be a k × 1 vector with k being the length of Z_t. The sample analogue of expression (5) is

$$(1/T)\sum_{t=0}^{T-1} h\left(R_{p,t+1},Z_t;\beta\right)=0. \tag{6}$$

Under standard regularity conditions on the utility function the estimation problem of the relevant parameters can be interpreted as a method of moments estimator as developed by Hansen (1982). The idea behind the Generalized Method of Moments (GMM) is to choose β so as to make the sample moment $(1/T)\sum_{t=0}^{T-1} h\left(R_{p,t+1},Z_t;\beta\right)$ as close to zero as possible. This is achieved by minimizing the scalar

$$\left[(1/T)\sum_{t=0}^{T-1} h\left(R_{p,t+1},Z_t;\beta\right)\right]^{'} V_T^{-1}\left[(1/T)\sum_{t=0}^{T-1} h\left(R_{p,t+1},Z_t;\beta\right)\right], \tag{7}$$

where V_T admits different choices of the covariance matrix. In a first stage V_T is the identity matrix and in a second stage, to gain efficiency, this matrix is replaced by a consistent estimator of the asymptotic covariance matrix, V, of the random vector $h\left(R_{p,t+1}, Z_t; \overline{\beta}\right)$. To find a suitable expression for this estimator we exploit condition (4) that implies that $h\left(R_{p,t+1}, Z_t; \beta\right)$ is a martingale difference sequence with respect to Ω_t. Using this fact, V_T can be expressed as

$$V_T = (1/T) \sum_{t=0}^{T-1} h\left(R_{p,t+1}, Z_t; \overline{\beta}\right) h'\left(R_{p,t+1}, Z_t; \overline{\beta}\right), \tag{8}$$

with $\overline{\beta}$ a consistent estimator of β obtained from minimizing (7) in the first stage. Asymptotic inference on these coefficients is obtained using standard results on GMM estimation. Thus, the asymptotic covariance matrix of the GMM estimator vector for β is

$$\Gamma_T = (1/T) \left[G_T' V_T^{-1} G_T \right]^{-1} \tag{9}$$

where

$$G_T = (1/T) \sum_{t=0}^{T-1} \frac{\partial h\left(R_{p,t+1}, Z_t; \overline{\beta}\right)}{\partial \beta} \tag{10}$$

In order to make these theoretical results operational we assume that the investor's utility function is isoelastic or CRRA and takes the following form:

$$U\left(R_{p,t+1}\right) = \frac{(1+R_{p,t+1})^{1-\gamma}}{1-\gamma} \tag{11}$$

with γ the investor's constant relative risk aversion (CRRA) coefficient. If $\gamma=1$ the utility function is $U(R_{p,t+1}) = \log\ (1+R_{p,t+1})$. The choice of this family of utility functions is standard in portfolio theory problems and asset pricing; see Brandt (1999) and references therein.

2. 3 Portfolio performance measures

We use the following metrics to measure the economic performance of the portfolios: 1) the Sharpe ratio, calculated as the mean portfolio excess return divided by the portfolio return volatility, 2) the Sortino ratio, calculated as the average period return in excess of the target return, which is the risk-free rate, divided by the target downside deviation, and 3) the difference in certainty equivalent returns (CERs), defined as the annualized difference between the CER calculated from the utility of the models that incorporate the state variables and the CER corresponding to the utility using the naïve size factor strategy. The CER is in this context a guaranteed return that makes the investor indifferent in expected terms between the risky portfolio $R_{p,t+1}$ and the riskless strategy paying off CER. Under CRRA utility, the CER is computed as:

$$CER = \left((1-\gamma) T^{-1} \sum_{t=1}^{T} U\left(1+R_{p,t+1}\right) \right)^{1/(1-\gamma)} - 1. \tag{12}$$

3. Choice of state variables

In our dynamic setting, in order to be able to determine the timing for reversing the investment positions between the risk-free asset and the SMB factor, it is crucial to be able to predict the dynamics of the latter asset. To do this we need to understand the variables that exhibit predictive power for the dynamics of the returns on such SMB factor.

Theoretically, the presence of credit market imperfections, and more specifically, the existence of lending agency costs have larger negative effects on the economic performance of small firms than of large firms, see the seminal article by Gertler and Gilchrist (1994). This negative effect is reflected by the presence of a higher borrowing premium on external financing by small firms than large firms. During recessions the underlying risk aversion increases and augments the cost of borrowing by the private sector. Whereas large firms can seek for alternative ways of financing during these periods, small firms still need to rely on the standard credit channels and can suffer more from adverse credit conditions. This effect is likely to reduce production and profits for small firms compared to large firms. The stance on monetary policy also has asymmetric effects in terms of firm size. Thus, it is expected that monetary policy tightening affects more adversely small firms than large firms by dampening their access to credit. The relationship between monetary policy tightening and an impoverishment in credit market conditions is also discussed by Bernanke et al. (1999). Small and large firms also differ on their level of liquidity. This factor gains importance during recession periods characterized by a drain in liquidity that is more relevant for small firms than large firms. During these periods investors require a higher expected return for small firms than for large firms to compensate for potential liquidity shortages under financial distress, see Amihud (2002). Vassalou (2004) also suggests that small cap firms have higher default risk than large firms. This author actually argues that the size effect could be driven by the firms with the smallest capitalization within the small caps group.

The above theories on credit market imperfections are empirically captured by the model proposed by Quirós and Timmermann (2000). These authors propose a two-state Markov Regime Switching model for the conditional mean excess return on equity portfolios classified according to the size factor. In this nonlinear model the conditional mean excess return depends on the one month Treasury-bill rate, the default premium, the annual rate of growth of the monetary base and the dividend yield. The transition probability between states is determined by the annual rate of growth of the Composite Index of Leading Indicators. Quirós and Timmermann (2000) find conditional Sharpe ratios that increase during recessions and fall in the ensuing expansion states, allowing them to implement simple profitable investment rules. The empirical work by these authors illustrates the importance of aggregate credit and monetary policy variables for explaining the dynamics of the size sorted firms' Sharpe ratio. The predictive power of the above variables is also supported by Petkova (2006). This author shows that the success of the SMB factor at describing the cross section of stock returns is partly due to the correlation between the factor and the innovations in variables connected to the investment opportunity set. These variables include the Treasury-bill yield, the default spread, the term spread and the dividend yield.

Another important channel for evaluating the dynamics of size sorted firms is through investor sentiment variables. Baker and Wurgler (2006) show that periods of high investor sentiment predict future underperformance of small caps vs. large caps, suggesting that in those periods the small caps, which are more difficult to value and arbitrage than large caps, are likely to be overpriced. Kurov (2010) shows that investor sentiment is also affected by the stance on monetary policy, especially during bear markets and recessions.

The above discussion on the choice of relevant variables to predict the SMB factor highlights the role of variables related to interest rates, overall credit conditions, market risk aversion and economic sentiment. Interest rates are represented by the level and slope factors of a European yield curve obtained from German data on sovereign bonds. The level and slope factors of the German government yield curve are expected to convey information about financing conditions in Europe, the business cycle outlook and the underlying risk premia in bond markets, which is linked to the external funding premium faced by firms. Extant financial conditions are captured in our model by the momentum in the level factor. Market risk aversion is captured by financial indexes such as the three month change of the VIX, often referred to as the fear gauge, and the three month change of the CRB Commodity index. Finally, we consider the survey-based German Economic Sentiment Indicator (ESI) to capture economic agents' beliefs about current and future economic conditions. This indicator is made up of five sectorial confidence indicators with different weights: Industrial confidence indicator (40%), Services confidence indicator (30%), Consumer confidence indicator (20%), Construction confidence indicator (5%) and Retail trade confidence indicator (5%). Confidence indicators are arithmetic means of seasonally adjusted balances of answers to a selection of questions closely related to the reference variable they are supposed to track. The ESI is calculated as an index with mean value of 100 and standard deviation of 10 over a fixed standardised sample period. Gelper and Croux (2010) illustrate the predictive power of this indicator for the short term Euro area growth. We hypothesize that the German ESI may also contain sufficient economic sentiment information to have power to predict the relative valuation between small and large caps portfolios.

It follows from these arguments that our proposed specification for $\alpha_{SMB,t}$ is:

$$\alpha_{SMB,t} = \alpha(Z_t;\beta) = \beta_0 + \beta_{Level}\, Level_t + \beta_{Slope}\, Slope_t + \beta_{Level\,Mom}\, Level\,Mom_t + \beta_{CVIX}\, CVIX_t + \beta_{CRB}\, CRB_t + \beta_{ESI}\, ESI_t. \qquad (13)$$

4. Empirical results

This section assesses the economic performance of the SMB factor as a naïve investment portfolio and compares it against the dynamic version that also considers a risk-free asset and the possibility of time-varying weights allocated to each asset return. The latter portfolio can be regarded as an investment instrument offering potential improvements upon the naive strategy during recessions and financial turmoil periods.

The dynamic weights are driven by a set of state variables proxying macroeconomic and financial conditions.

4.1. Data description

Our data covers the period March 1993 to November 2013. We collect monthly return data from Bloomberg on the Eurostoxx 50 and the Stoxx Europe Small 200. We consider the Eurostoxx 50, which is a market capitalization-weighted stock index of 50 large European companies operating within the Euro zone nations, as representative of European large caps. Similarly, the Stoxx Europe Small 200 is chosen as representative of small capitalization companies in Europe[1]. This index covers Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom. Monthly data are also collected from Bloomberg on the German three-month Treasury bill, the VIX index and the CRB Industrial commodity index.

We extract the level and slope factors of the German sovereign interest rates by applying principal components analysis to the term structure of monthly zero-coupon yields available at Deutsche Bundesbank. The first principal component explains 97.08% of the total variance, the share of variance explained by the first two factors is 99.85% and 99.98% if we include the third factor. The principal component loadings are reported in Figure 1. Data on the U.S. Baa-Aaa yield spread are obtained from the U.S. Federal Reserve. The ESI is provided by the European Commission. Figure 2 plots the state variables for the period under consideration. We demean and standardize all the state variables in the optimization process.

[1] It is important to notice that data on the Eurostoxx 50 and the Stoxx Europe Small 200 futures are available for trading and allow the straightforward implementation of the investment strategies proposed in this paper.

[Insert figure 1 and 2 about here]

Table 1 shows the cross-correlations between the period t state variables and the naïve investment strategy return on the SMB factor in excess of the German three-month Treasury bill rate at period $t+1$. Most of the correlations are modest with the exception of the negative correlation between the level factor of interest rates and the size factor, and to a lesser degree between the level momentum and the three-month change of the VIX and CRB indexes.

[Insert table 1 about here]

4.2. Empirical Portfolio Performance

We study the empirical performance of the optimal European size factor strategy discussed in preceding sections. The section also discusses the robustness of the findings to variations of the basic model; in particular, we contemplate the following robustness measures: First, we replace the German ESI by the Euro area ESI, and we consider separately the U.S. Baa-Aaa yield spread, the one-month change in the U.S. Baa-Aaa yield spread and the one-month change in the slope factor of interest rates. Second, we study the effect of using instead the definition of European small and large caps provided in Kenneth French's database. As a final robustness measure, we carry out an out-of-sample exercise to provide further support to the in-sample analysis.

4.2.1 In-sample results

Table 2 presents the in-sample parameter estimates optimized for a power utility function defined by the CRRA parameters $\gamma = (2, 5, 10, 40, 100)$. The econometric estimation is performed using a two-step estimator and a weighting matrix that accommodates the presence of heteroskedasticity and autocorrelation up to four lags using the Bartlett kernel. This exercise aims to ascertain the robustness of the results across different degrees of investor risk aversion.

The sign pattern is consistent across gamma, with increasing estimates of the beta coefficients associated with decreasing values of gamma, revealing an inverse relationship between the degree of investor risk aversion and its responsiveness to changes in the information set. The results provide evidence that the optimal portfolio weight allocated to the European size factor is negatively related to the level factor of European interest rates, the three-month change of the level factor of European interest rates and the German ESI. As expected, an environment of high interest rates and a positive momentum of the level factor affect more adversely small caps, making the portfolio weight allocated to the European size factor decrease. Interestingly, high values of the ESI variable also have a negative influence on the optimal portfolio allocation to the European size factor. We interpret this finding as evidence of the existence of two different transmission channels between economic sentiment and the relative performance of small firms compared to large firms. One channel is through the larger negative impact of increases in interest rates on investors' views on small firms' returns due to tighter financing conditions; and a second related channel is through the impact that tight monetary policies have on investors' valuations of small cap stocks usually leading to downward corrections in market prices and lower returns. This result would be in the spirit of Baker and Wurgler (2006).

Table 2 also reveals that the portfolio weight allocated to the European size factor is negatively related to the three-month change of the VIX but positively related to the three month change of the CRB Industrial return. An interpretation of these results is that increasing levels of market risk aversion corresponded by a rise in market risk premiums, reflected by spikes of the VIX and the plunge of the commodity index, can trigger a flight to quality from small caps to large caps, implying negative returns of the SMB factor during these periods.

Table 3 reports the investment performance of optimized strategies in-the-sample vs. the passive strategy. The strategy that uses all of the state variables achieves a Sharpe ratio of 1.40, which is 222 points greater than the passive SMB Sharpe ratio that obtains a negative value over the whole evaluation period. The dynamic strategy also yields a portfolio return distribution characterized by a positive skewness and low kurtosis, reducing, in turn, the likelihood of crash risk. The optimal strategy delivers a Sortino ratio of 3.10 in contrast to -0.98 obtained by the naive SMB portfolio. These results suggest that the possibility of switching between long and short positions on the SMB factor provides investors protection against the occurrence of downside risk. The in-sample certainty equivalent of the optimized strategy that considers all the variables almost reaches an impressive 32% per year higher than that of the naive strategy, giving further support to the economic significance of the results.

Figure 3 reports the optimal dynamic weights allocated to the SMB factor during the in-sample period. The chart reveals the existence of four regimes. From February 1993 until December 2002 the optimal strategy is given by a short position on the SMB factor, taking advantage of the outperformance of large caps over small caps during this period. This episode corresponds to the period with the highest level of interest rates in our sample and is characterized by the late-1990's bubble in technological stocks that ended in the global economic recession lasting until the end of 2001 and the subsequent 2002 bear stock market. The second regime corresponds to the economic expansion fuelled by a global dovish monetary policy observed during the period February 2003 to February 2006. The weight allocated to the European size factor gains importance during this period. Interestingly, during this period the dynamic portfolio switches between long and short positions on the SMB factor possibly reflecting the uncertainty in financial markets extant during that period. The third period, from February 2006 to February 2009, corresponds with the final part of the subprime crisis episode and is characterized by investors' flight to quality from small cap stocks to large cap stocks. From February 2009 until the end of the sample, the optimal portfolio weight allocates most wealth to the European size factor. There are some exceptions at the beginning of this period corresponding with the European sovereign debt crisis. These turbulences in financial markets are captured by the optimal dynamic portfolio by taking a short position on the SMB factor during these episodes. In the remaining subsample characterizing this regime we observe how the dynamic portfolio tracks again the SMB factor illustrating the good performance of this factor in economic expansion periods and low interest rate scenarios. A closer look to the dynamics of the state variables reveals the importance of the level factor of the German sovereign term structure of interest rates in determining the dynamics of the optimal weights. This variable shows a declining pattern over the sample being especially important during the period from 2002 onwards. Negative values of the German ESI also correspond to long positions on the SMB factor and may be indicative of the presence of turning points in the small vs. large caps relative valuation, suggesting subsequent corrections. This result would be in the spirit of Baker and Wurgler (2006).

[Insert figure 3 about here]

Figure 4 plots the in-sample cumulative excess returns of the optimal European size factor vs. the passive SMB strategy, assuming a CRRA coefficient equal to five for the investor utility function. As can be readily seen from the plot, the returns on the dynamic size factor clearly dominate the returns on the passive strategy.

[Insert figure 4 about here]

4.2.2 Robustness

The first robustness exercise consists on including new state variables in the investor information set that could potentially predict the dynamics of the European size factor and its impact on the investor marginal utility through its relationship with the overall financing conditions and economic outlook. By doing so, we want to check the stability of the parameter signs and their magnitude, and the possibility of improving investment performance.

We consider the European Union Economic Sentiment Index (EU ESI) instead of the German ESI. Data for constructing the EU ESI are based on surveys carried out in all the member states of the European Union. We also include the U.S. Baa/Aaa spread in the investor information set. This spread mirrors credit conditions and is defined as the difference between the yields on Baa and Aaa rated corporate bond portfolios. The choice of this variable as opposed to its European counterparty is due to its availability. This variable is countercyclical as noted by Fama ad French (1989) and expected to proxy the increase in default risk during recessions. Its relation to the size factor is through the differences in rating scores between small and large firms that is reflected in larger differences in borrowing costs. We also consider the one month change in the U.S. Baa-Aaa yield spread and a variable reflecting the term or slope momentum, the one-month change in the slope factor of interest rates. The choice of these variables is justified by Hann and Lee (2006), that find that changes in the default spread and the term spread capture systematic differences in average returns along the size and book-to-market dimensions, making the SMB and HML factors superfluous in explaining the cross-section of portfolios returns sorted on firm size and book-to-market. Intuitively, the increase of the default spread signals an expected deterioration of credit market conditions with stronger negative effects on the small caps that are more vulnerable to changing credit market conditions, see Quiros and Timmermann (2000). Increases of the term spread usually anticipate higher future interest rates controlling for the presence of a bond risk premium that reach their highest values at the trough of the business cycle. The combination of both effects could affect more negatively the performance of small caps compared to large caps in the short run.

The second robustness exercise considers a different European size portfolio as an alternative benchmark to the one discussed above. Thus, we implement the optimal European size factor strategy using Kenneth French's database available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 4 shows that including the European Union ESI instead of the German ESI in the investor's information set provides similar results, but with a slightly lower statistical significance of the estimated parameters. The economic value of the optimized investment portfolio remains, however, almost unaltered. The Sharpe ratio decreases to 1.32, the excess certainty equivalent of the optimized strategy reaches values around 30% and the Sortino ratio is 2.86. These values are consistent with previous findings and suggest, as before, the good performance of the optimized investment strategy when the SMB factor performs poorly. Our preference for the German ESI, which is likely to be subject to fewer revisions than the European Union ESI, is due not only to its higher simplicity but also its positive effect on investment performance.

[Insert table 4 about here]

The results in Table 4 also show that including the U.S. Baa-Aaa spread from the set of state variables hardly affects the parameter estimates of the rest of state variables in the GMM exercise and the comparison in terms of economic significance between portfolio performances. More importantly, the U.S. Baa-Aaa yield spread is not statistically significant; the information carried by this variable seems to be subsumed under the other state variables. We reach the same conclusions using the U.S. Baa-Aaa yield spread momentum and the slope momentum, which has a negative marginal effect on the optimal weight allocated to the European size factor (p-value equals to 0.11). In the light of these results, we maintain the preference for our initial specification of the investor information set.

Table 5 provides the optimization results using Kenneth French's database. The European SMB factor delivers a -7.64% excess return in annualized terms in our sample and a Sortino ratio of -1.05. The optimal European size factor is again negatively related to the level factor of interest rates, the German ESI, the three month change of the VIX, and positively related to the CRB Industrial return. The economic value of the optimal investment portfolio, which is highly influenced by the level factor of interest rates, the German ESI and the three month change of the VIX is characterized by a Sharpe ratio around 0.80, an excess equivalent return larger than 15% and a Sortino ratio of 1.33.

[Insert table 5 about here]

4.2.3. Out-of-sample results

In this exercise the optimal portfolio is re-estimated on a monthly basis using a rolling window of data until the end of the sample. The investor uses the information available up to period t, reflected in the values of the state variables, to estimate the dynamic weight function defining the optimal portfolio between t and t+12. The first portfolio is computed with data from March 1993 to December 2002. The outstanding performance of the size portfolio over our out-of-sample period spanning from January 2003 constitutes an excellent framework for testing the out-of-sample performance of the dynamic strategy in comparison with the naïve SMB benchmark. For the overall out-of-sample period the static SMB portfolio provides an excess return of 6.67%, a Sharpe ratio of 0.73 and a Sortino ratio larger than 1.

In order to be able to compare the naïve and dynamic strategies in terms of excess returns, and abstract from the level of volatility underlying each of them we take as a point of reference the volatility of the naïve European size factor and construct our optimal portfolio such that its volatility matches the ex post volatility of SMB. This assumption imposes further restrictions on the level of leverage assumed by the investor and reflected in further constraints on the size of the weight function. Table 6 shows the remarkable performance of the dynamic strategy out of sample. The average optimal weight is 0.57, the strategy's Sharpe ratio is 0.94, which is 0.21 units more than under the SMB strategy. The Sortino ratio attains a value of 1.79 compared to a value of 1.08 for the passive strategy, and the out-of-sample certainty equivalent of the optimized strategy is about 92 basis points per year higher than for the naïve SMB factor.

[Insert table 6 about here]

Figure 5 plots the out-of-sample performance of the optimal European size factor vs. the naïve SMB. As can be readily seen from the plot, the returns on our optimal portfolio dominate those of the naïve strategy in almost the entire out-of-sample period. Investing 1 Euro in the SMB factor portfolio would yield 1.47 Euros at the end of the period, whereas the dynamic strategy would produce a terminal wealth of 2.29 Euros.

[Insert figure 5 about here]

Figure 6 plots the dynamics of the optimal European size factor over the period January 2003 to September 2013. This graph suggests that an optimal strategy is to have a short position on the SMB factor during the period 2007-2009 and a long position afterwards. These findings can be rationalized by noting that the first period is characterized by turmoil in financial markets and increases in market risk aversion triggered by the subprime crisis. The second period reflects, however, the higher normalization of financial markets and the low level of interest rates.

[Insert figure 6 about here]

5. Conclusion

In this paper we discuss the choice of a European size factor as a suitable investment style and address the macroeconomic and financial determinants with power to explain its dynamics. These variables and the inclusion of a risk-free asset in the investment opportunity set allow us to derive an optimal investment portfolio given by a long position on the European size factor in periods of economic expansion and a short position in periods of economic downturn and financial turbulence that is compensated by a long position in the risk-free asset. Our empirical analysis also uncovers the existence of a negative relationship between the returns on the size factor, the level factor of interest rates, market risk aversion and the German Economic Sentiment Index reported by the European Commission. The rationale behind these relationships is the asymmetric effect of financial turmoil and tight monetary policies in small and large capitalization firms reflected in increases in interest rates, price corrections of overvalued small firms and tighter borrowing and financing conditions. Our results are meaningful from the perspective of investor's welfare. An investor who follows the optimized dynamic European size factor strategy attains a higher Sharpe ratio, Sortino ratio and certainty equivalent than under the passive strategy. The latter portfolio is, nevertheless, an interesting strategy during economic expansions.

References

Ait-Sahalia, Y.,& Brandt, M. (2001). Variable selection for portfolio choice. *Journal of Finance*, *56*, 1297–1351.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5, 31-56.

Baker, M., and Jeffrey W., (2006). Investor Sentiment and the Cross Section of Stock Returns, *Journal of Finance*, 61 (4), 1645-1680.

Bernanke, B., Gertler, M., Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. Handbook of Macroeconomics, in: J. B.Taylor and M.

Brandt, M. (1999). Estimating portfolio and consumption choice: A conditional method of moments approach. *Journal of Finance*, *54*, 1609–46.

Brandt, M., Santa Clara, P., & Valkanov, R. (2009). Parametric portfolio policies exploiting the characteristics in the cross section of equity returns. *Review of Financial Studies*,*22*, 3411–3447.

Fama, E., & French, K. (1993). Common risk factors in the returns on stock and bonds. *Journal of Financial Economics*, 33, 3–56.

Fama, E. and K. French, (1996). Multifactor explanations of asset pricing anomalies, *Journal of Finance*, 51, 55-84.

Gelper, S., Croux, C. (2010). On the construction of the European Economic Sentiment Index. *Oxford Bulletin of Economics and Statistics*, 72, 47-62.

Gertler, M., Gilchrist, S. (1994). Monetary policy, business cycles and the behaviour of small manufacturing firms. *Quarterly Journal of Economics*, 109, 309-340.

Hann, J., Lee, H. (2006). Yield spreads as alternative risk factors for size and book-to-market. Journal of Financial and Quantitative Analysis, 41, 2, 245-269.

Hansen, L. (1982). Large sample properties of generalized method of moments estimators. *Econometrica*, *50*, 1029–1054.

Kapadia, H. (2011). Tracking down distress risk. *Journal of Financial Economics*, 102, 167-182.

Kurov, A. (2010). Investor sentiment and the stock market's reaction to monetary policy. *Journal of Banking and Finance*, 34, 139-149.

Kyotaki, N., Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105, 211-248.

Quirós, G., Timmermann, A. (2000). Firm size and cyclical variation in stock returns. *Journal of Finance*, 55, 1229-1262.

Liew, J., Vassalou, M. (2000). Can book-to-market, size and momentum be risk factors that predict economic growth? *Journal of Financial Economics*, 57, 221-245.

Petkova, R. (2006). Do the Fama-French factors proxy for innovation in predictive variables?. *Journal of Finance*, LXI(2), 581-612.

Vassalou, M., Xing, Y. (2004). Default risk in equity returns. *The Journal of Finance*, LIX (2), 831-868.

Table 1: Correlation matrix

	$R^e_{SMB,t+1}$	$Level_t$	$Slope_t$	Level $Momentum_t$	$CVIX_t$	CRB_t	ESI_t
$R^e_{SMB,t+1}$	1						
$Level_t$	-0.33	1					
$Slope_t$	0.03	0.03	1				
Level $Momentum_t$	-0.03	0.14	-0.09	1			
$CVIX_t$	-0.18	0.07	-0.07	-0.26	1		
CRB_t	0.14	0.04	0.04	0.48	-0.35	1	
ESI_t	-0.18	0.01	-0.24	0.17	0.06	0.04	1

This table presents the correlation matrix of the investment strategy return, that consists of going long in the European small caps and short in the European large caps (SMB), in excess of the German three-month Treasury bill rate at period *t+1*, and the state variables at period *t*: the level factor of European interest rates (Level), the slope factor of European interest rates (Slope), the three-month change of the level factor of European interest rates (Level Momentum), the three-month change of VIX index (CVIX), the three-month CRB Industrial return (CRB), and the Germany economic sentiment indicator (ESI). The return horizon is one month.

Table 2: Simple linear portfolio policy. In-sample results

Variable	CRRA=2	CRRA=5	CRRA=10	CRRA=40	CRRA=100
β_{Level}	-7.13	-3.46	-1.79	-0.44	-0.16
t-stat	(-4.90)	(-4.91)	(-4.98)	(-4.94)	(-4.64)
β_{Slope}	1.58	0.44	0.18	0.05	0.03
t-stat	(1.02)	(0.52)	(0.41)	(0.49)	(0.84)
$\beta_{Level\ Momentum}$	-11.25	-7.13	-3.80	-1.00	-0.43
t-stat	(-1.96)	(-2.13)	(-2.17)	(-2.31)	(-2.60)
β_{CVIX}	-3.69	-1.80	-0.93	-0.24	-0.10
t-stat	(-2.91)	(-3.13)	(-3.19)	(-3.23)	(-3.17)
β_{CRB}	2.77	1.50	0.79	0.20	0.10
t-stat	(2.45)	(2.78)	(2.85)	(2.99)	(3.13)
β_{ESI}	-2.84	-1.25	-0.64	-0.16	-0.06
t-stat	(-2.92)	(-2.87)	(-2.89)	(-2.90)	(-2.64)

This table shows estimates of the optimal investment strategy policy, which consists of going long in the European small caps and short in the European large caps (SMB), specified in equation (13) and optimized for a power utility function with different CRRA coefficients (γ=2,5,10,40,100) using these state variables: the level factor of European interest rates (Level), the slope factor of European interest rates (Slope), the three-month change of the level factor of European interest rates (Level Momentum), the three-month change of VIX index (CVIX), the three-month CRB Industrial return (CRB), and the Germany economic sentiment indicator (ESI).

Table 3: Investment performance of the optimized strategies in-the-sample

Strategy	Mean	Skewness	Kurtosis	SR	Sortino ratio	CER	Mean α	STD α
SMB	-8.87	0.05	0.15	-0.82	-0.98			
Level, Slope, Level Momentum, CVIX, CRB, ESI (all in)	16.50	.0.61	1.47	1.40	3.10	31.57	-0.39	1.13

This table reports the in-sample performance of the optimized investment strategies in the period 1993:03 to 2013:09. The optimizations use a power utility with a constant risk aversion parameter equal to 5. The naïve investment strategy return displays statistics for the strategy of going long in the European small caps and short in the European large caps (SMB). SR is the optimized portfolio's Sharpe ratio; CER measures the difference in the annualized certainty equivalent of each strategy vs. the naïve SMB size portfolio. The Sharpe ratios and Sortino ratios are annualized; the mean excess return is also annualized. The mean α is the average optimal weight allocated to the SMB factor and STD α is its standard deviation.

Table 4: Simple linear portfolio policy. In-sample results considering EU ESI, and the U.S. Baa-Aaa yield spread, the one-month change in the U.S. Baa-Aaa yield spread, and the one-month change in the slope factor.

Variable	CRRA=5	CRRA=5	CRRA=5	CRRA=5	SMB
β_{Level}	-3.02	-3.03	-3.46	-3.88	
t-stat	(-4.44)	(-2.92)	(-4.93)	(-5.27)	
β_{Slope}	0.43	0.41	0.42	0.05	
t-stat	(0.51)	(0.49)	(0.50)	(0.07)	
$\beta_{Level\ Momentum}$	-7.61	-7.33	-6.95	-7.27	
t-stat	(-2.29)	(-2.11)	(-2.09)	(-2.23)	
β_{CVIX}	-1.72	-1.78	-1.76	-1.73	
t-stat	(-2.93)	(-3.11)	(-3.12)	(-2.92)	
β_{CRB}	1.53	1.71	1.72	1.66	
t-stat	(2.77)	(2.44)	(2.13)	(3.11)	
$\beta_{EU\ ESI}$	-0.91				
t-stat	(-2.14)				
β_{ESI}		-0.95	-1.20	-1.43	
t-stat		(-1.89)	(-2.85)	(-3.02)	
$\beta_{Spread\ Baa/Aaa}$		0.54			
t-stat		(0.67)			
$\beta_{Spread\ Baa/Aaa\ Momentum}$			0.46		
t-stat			(0.42)		
$\beta_{Slope\ Momentum}$				-3.96	
t-stat				(-1.56)	
Mean excess return	15.76	16.48	16.06	16.31	-8.87
SR	1.32	1.36	1.35	1.38	-0.82
Sortino Ratio	2.86	2.97	2.89	2.98	-0.98
CER	29.29	30.62	29.92	30.90	

This table shows estimates of the optimal investment strategy policy, which consist of going long in the European small caps and short in the European large caps (SMB), specified in equation (13) and optimized for a power utility function with a CRRA coefficient (γ=5) using two different specifications. The firs specification includes the following state variables: the level factor of European interest rates (Level), the slope factor of European interest rates (Slope), the three-month change of the level factor of European interest rates (Momentum Level), the three-month change of VIX index (CVIX), the three-month CRB Industrial return (CRB), and the European economic sentiment indicator (EU ESI) instead of the German economic sentiment indicator (ESI). Other specifications also include the U.S. Baa-Aaa yield spread, the U.S. Baa-Aaa yield spread momentum and the slope factor momentum. The table also reports the mean excess returns, Sharpe ratio, Sortino ratio and the CER defined as the difference in the annualized certainty equivalent of the optimized strategy vs. the naïve investment strategy return (SMB).

Table 5: Simple linear portfolio policy. In-sample results considering French database

Variable	CRRA=5	French SMB
β_{Level}	-1.77	
t-stat	(-2.27)	
β_{Slope}	0.91	
t-stat	(1.1)	
$\beta_{Momentum\ Level}$	3.71	
t-stat	(0.85)	
β_{CVIX}	-2.40	
t-stat	(-2.73)	
β_{CRB}	0.27	
t-stat	(0.39)	
β_{ESI}	-0.43	
t-stat	(-2.67)	
Mean excess return	8.46	-7.64
SR	0.81	-0.91
Sortino Ratio	1.33	-1.05
CER	15.10	

This table shows estimates of the optimal investment strategy policy, which consists of going long in the European small caps and short in the European large caps using French database (French SMB), specified in equation (13) and optimized for a power utility function with CRRA coefficient=5 using the following state variables: the level factor of European interest rates (Level), the slope factor of European interest rates (Slope), the three-month change of the level factor of European interest rates (Level Momentum), the three-month change of VIX index (CVIX), the three-month CRB Industrial return (CRB), and the German- economic sentiment indicator (ESI). The table also reports the mean excess returns, Sharpe ratio, Sortino ratio and the CER defined as the difference in the annualized certainty equivalent of the optimized strategy strategy vs. the naïve investment strategy return (SMB).

Table 6: Investment performance of the optimized strategies out-of-sample

Strategy	**Mean**	**Skewness**	**Kurtosis**	**SR**	**Sortino ratio**	**CER**	**Mean α**	**STD α**
SMB	6.67	0.11	0.40	0.73	1.08			
Level, Slope, Spread BAA/AAA, CVIX, CRB, ESI (all in)	14.03	0.59	1.62	0.94	1.79	0.92	0.57	1.40

This table reports the out-of-sample performance of the optimized investment strategies in the period 2003:01 to 2013:09. The optimizations use a power utility with a constant risk aversion parameter equal to 5. We use data from March 1993 to December 2002 to estimate the first optimal parametric portfolio. After this, the model is re-estimated every year using expanding rolling window of data until the end of the sample. The investor uses the estimates in period t to form the optimal European size portfolio between t and t+1, given the observed realization at time t of the state variables The naïve investment strategy return displays statistics for the strategy of going long in the European small caps and short in the European large caps (SMB). SR is the optimized portfolio's Sharpe ratio; CER measures the difference in the annualized certainty equivalent of each strategy vs. the naïve currency carry trade. The Sharpe ratios and Sortino ratios are annualized; the mean excess return is also annualized. The mean α is the average optimal currency carry trade bet and STD α is its standard deviation.

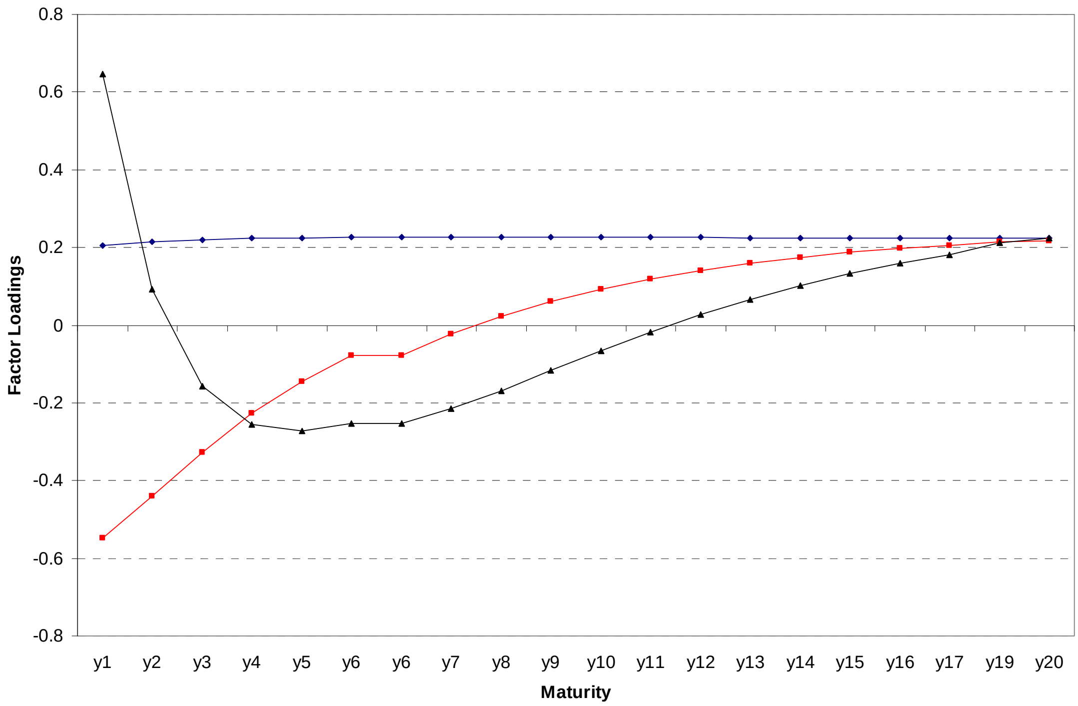
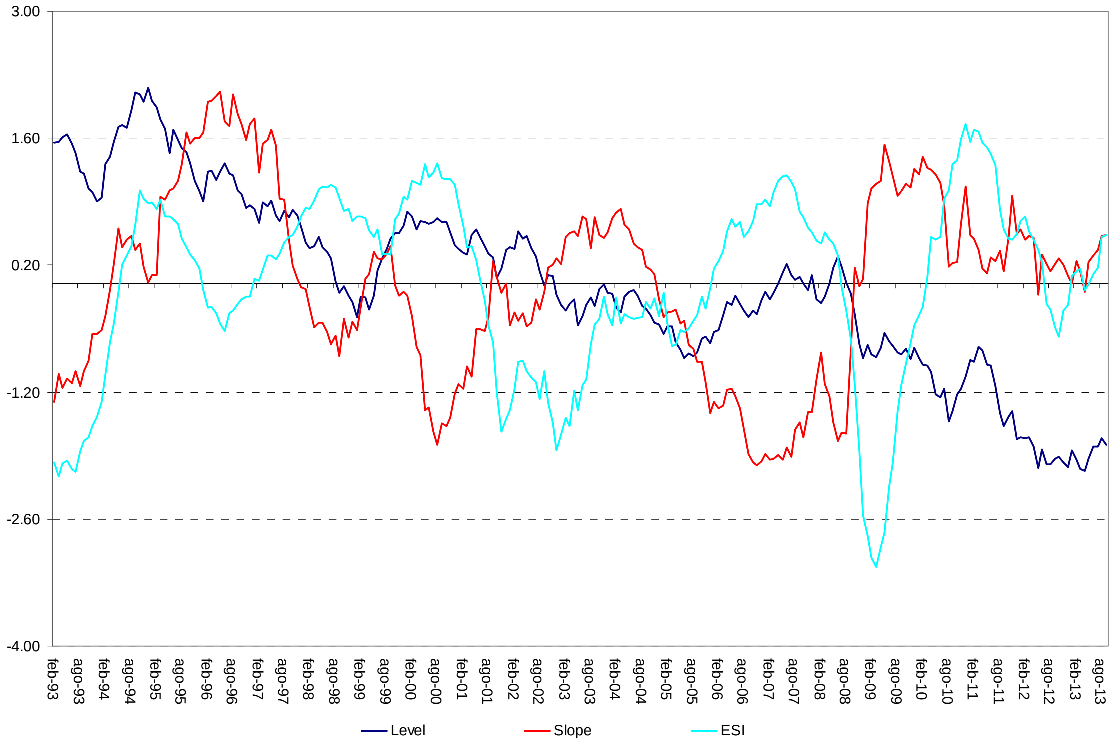


Figure.1: Three most important factor loadings driving the German zero-coupon yield curve.

This figure plots the three most important factor loadings driving the German zero-coupon yield curve. These factor loadings are obtained from the Principal Component analysis of monthly observations of the database of German zero-coupon yields available at Deutsche Bundesbank for the period March 1993 to September 2013.



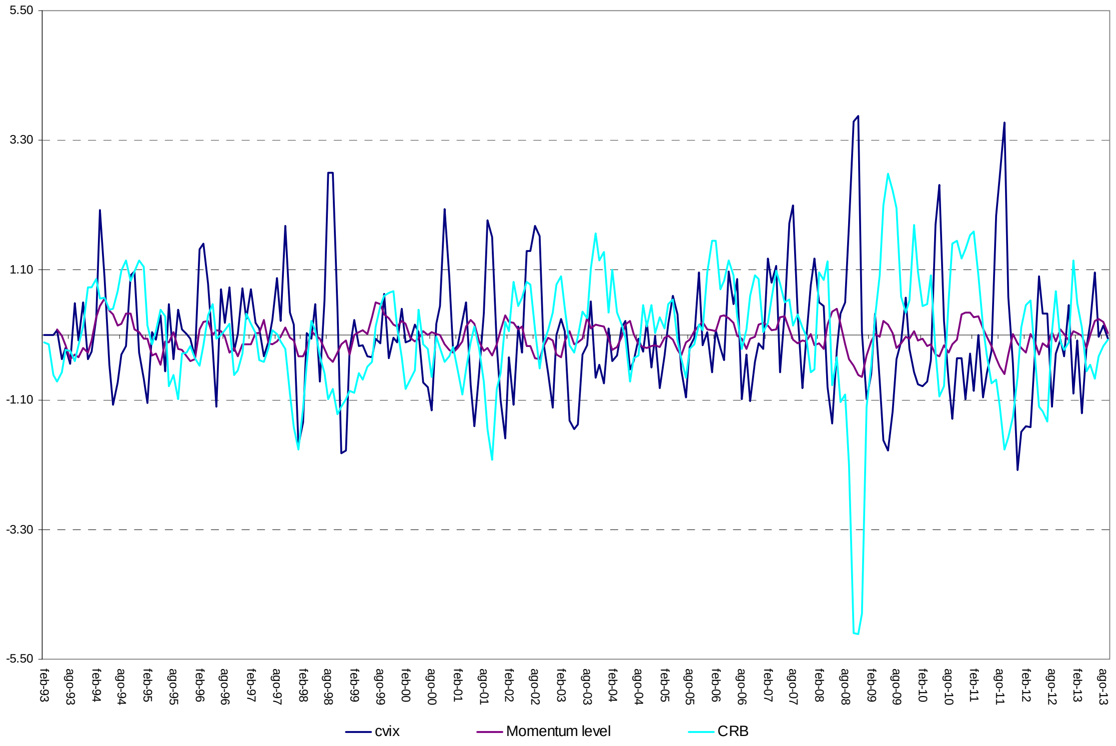


Fig.2: State variables dynamics

This figure plots the standardized across time state variables time series dynamics in the period from January 1993 to September 2013. The state variables are: the level factor of European interest rates (Level), the slope factor of European interest rates (Slope), the U.S.Baa-Aaa yield spread, the three month change of VIX index (CVIX), the three month CRB Industrial return (CRB) and the European economic sentiment indicator (ESI).

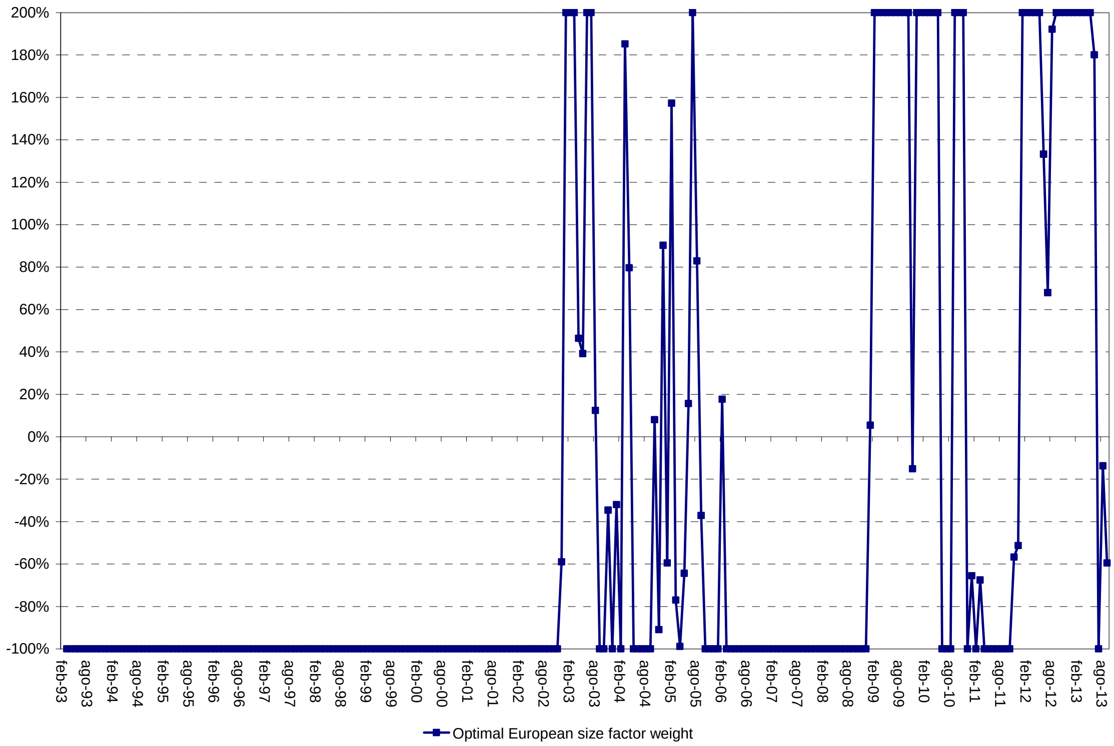


Fig.3: Optimal European size factor weight

This figure plots the in-sample optimal European size factor portfolio policy for a power utility function with a relative risk aversion parameter equal to 5 and a linear portfolio specification (see eqn. 13).

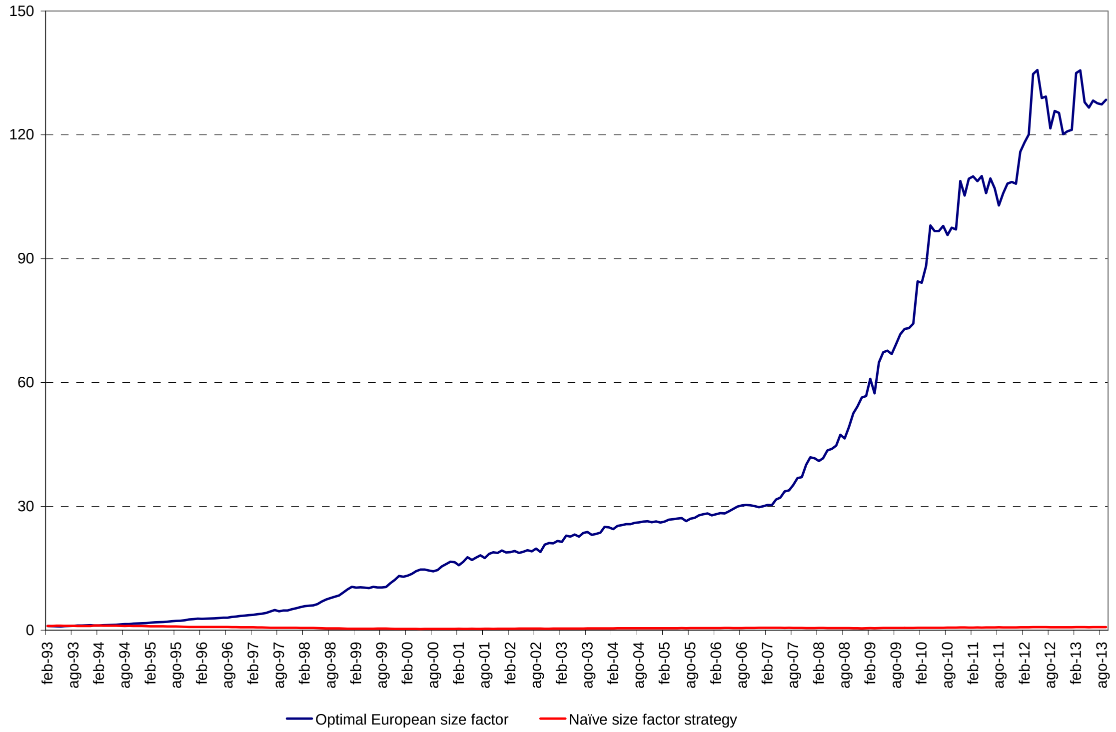


Fig.4: Evolution of in-sample cumulative excess returns

This figure plots the in-sample optimal European size factor and the naïve size factor cumulative excess returns. The optimal European size factor excess returns are computed for a power utility function with a relative risk aversion parameter equal to 5 and a linear portfolio specification (see eqn. 13).

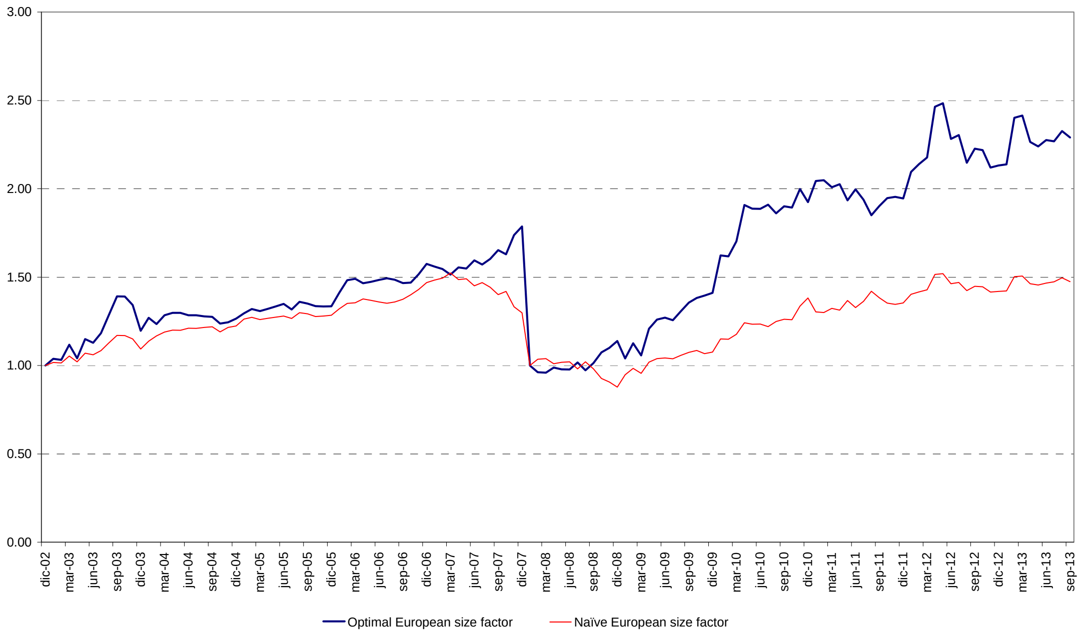


Fig.5: Evolution of out-of-sample cumulative excess returns

This figure plots the out-of-sample optimal European size factor and naïve European size factor cumulative excess returns. The optimal European size factor excess returns are computed for a power utility function with a relative risk aversion parameter equal to 5 and a linear portfolio specification (see eqn (13)). We use data from March 1993 to December 2002 to estimate the first optimal parametric portfolio. After this, the model is re-estimated every month using a rolling window of data until the end of the sample. The investor uses the estimates in period t to form the optimal European size portfolio between t and t+12, given the observed realization at time t of the state variables The naïve investment strategy return (SMB) displays statistics for the strategy of going long in the European small caps and short in the European large caps.

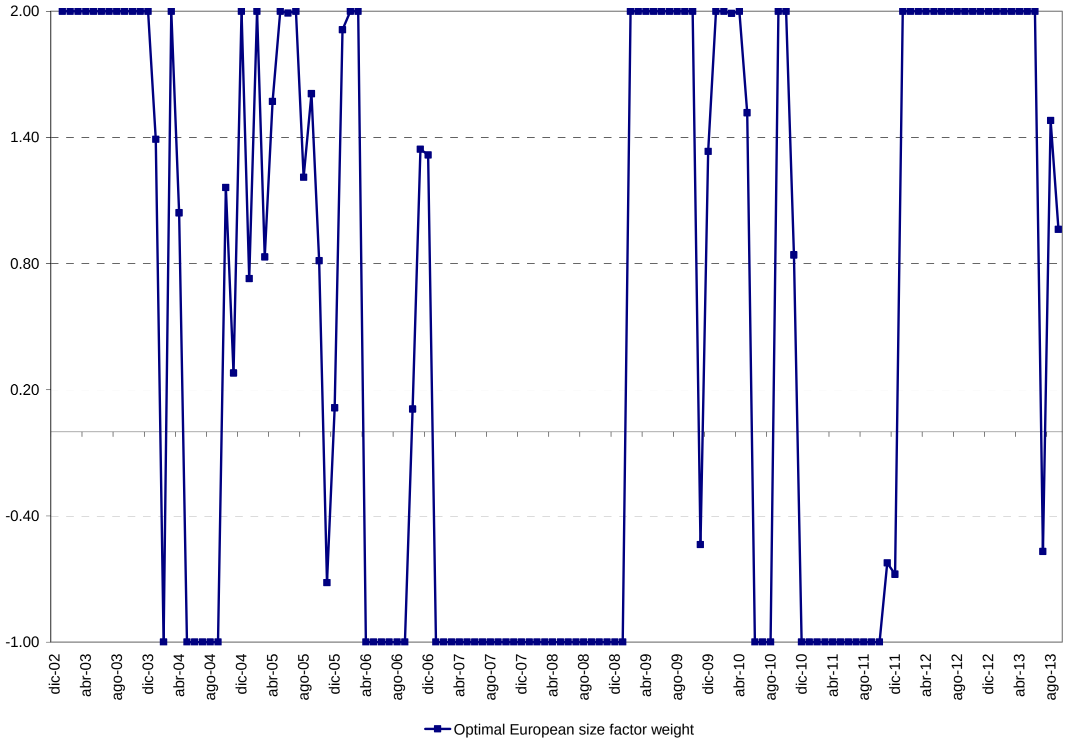


Fig. 6: Out-of-sample optimal European size factor

This figure shows the out-of-sample optimal European size factor portfolio policy for a power utility function with a relative risk aversion parameter equal to 5 and a linear portfolio specification (13) from January 2003 to September 2013.