

International diversification benefits using a multivariate VAR-GARCH approach

Abstract

The aim of this study is to provide empirical evidence of the international diversification benefits obtained employing not only time-varying volatility forecasts but also time-varying return forecasts from a multivariate VAR-GARCH model which considers the dynamic relationships in return series as well as in volatilities and correlations. To that end, instead of using market indexes from different investment areas, we employ Exchange Trade Funds actively traded on the New York Stock Exchange in recent years. It avoids non-synchronous problems as well as allowing us to allocate internationally on a daily basis for which this model is especially appropriate. Our overall results show that using this technique it is possible to obtain economic gains and out-perform the common benchmark strategies, even when the costs associated with the daily rebalance of each portfolio are taken into account. These findings are relevant not only for academics, but also for practitioners, especially for professional portfolio managers.

Keywords

Multivariate VAR-GARCH; Optimal diversification; Exchange Trade Funds; Performance evaluation.

JEL Classification: G10, G11, G14.

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1. Introduction

The classical theory of optimal portfolio selection proposed by Markowitz (1952) states that asset correlations have a significant impact on portfolio risk profile. For that reason, investors usually choose to diversify internationally and further reduce their portfolio risk assumptions. However, the question of whether or not international diversification benefits are still substantial in the current context of increasing market correlations has yet to be answered. Although this issue has attracted considerable attention in the financial literature over the last few years, previous empirical evidence provides mixed results. While Das and Uppal (2004) and Driessen and Laeven (2007), among others, find that diversification benefits have decreased considerably, Chue (2005) and Chiou (2009) suggest that international diversification benefits for US investors persist even when the international stock returns correlations are high.

Moreover, the practical application of Markowitz's (1952) portfolio theory requires implementing the expected return and the covariance matrix of the asset under consideration in the optimization programming problem. In this sense, the traditional approach has been based on computing the sample mean and covariance matrix of asset returns up to time t and uses them as the required inputs to the optimization program. However, this model-free approach produces extreme portfolio weights that fluctuate substantially over time and perform poorly in an out-of-sample period as documented by Hodges and Brealey (1972), Michaud (1989), Best and Grauer (1991) and Litterman (2003), among others.

Nowadays, the criticism of Markowitz's practical implementation persists. More precisely, DeMiguel *et al.* (2009) document that it is not possible to beat the naïve strategy using the mean-variance portfolio diversification. More precisely, they show that for the monthly frequency the asset allocation errors of the naïve rule are smaller than those of the optimization one. However, Kirby and Ostdiek (2012) show that the results of DeMiguel *et al.* (2009) are conditioned by their target expected return. On the other hand, Kirby and

Ostdiek (2012) show that when the mean-variance approach is implemented by targeting the conditional expected return of the naïve strategy it is possible to beat the naïve diversification for most of the monthly data sets considered in the work of DeMiguel *et al.* (2009), although their results are not robust to the presence of transaction costs.

In another strand of financial literature, diverse authors have proposed various multivariate GARCH specification to model the dynamic dependence structure of multivariate time series and, more precisely, to parameterize the dynamic equation of the conditional covariance. One of the most popular multivariate specifications is the BEKK model, introduced by Baba, Engle, Kraft and Kroner (1991). Although it has been widely used in previous years, in practice it suffers from a number of problems.¹ An alternative multivariate GARCH model that overcomes these computational issues is the Dynamic Conditional Correlation (DCC) model proposed by Engle (2002) which describes the correlation dynamics among different asset classes and markets.

This line of research has important implications for investors because it gives them the opportunity to solve the classical allocation problem with forward-looking correlation forecasts obtained from these dynamic correlation models. Some examples of these empirical applications are those of Engle and Colacito (2006) and Jondeau and Rockinger (2012). However, these previous works have only analyzed the implications of volatility timing on international diversification decisions and have avoided the prediction of expected returns.

The main contribution of this study is to reflect the economic value of employing not only time-varying volatility forecasts but also time-varying return forecasts from a multivariate VAR-GARCH model which considers the dynamic relationships in return series as well as in volatilities and correlations for international diversification purposes.

Moreover, instead of using market indexes from different investment areas with the consequent non-synchronous problem, we employ Exchange Trade Funds (ETFs) actively traded on the New York Stock Exchange (NYSE) in recent years. It allows us to allocate

¹ It is computationally arduous because it involves the estimation of a large number of parameters. Consequently, multi-step ahead forecasts require a laborious effort.

internationally on a daily basis for which multivariate VAR-GARCH models are especially appropriate.

We primarily consider for our empirical research three ETFs which track the most relevant stock markets indexes of the US, UK and Japan: the Standard and Poor's Depositary Receipt as well as the Morgan Stanley Capital International iShares for the United Kingdom and Japan.² Therefore, we analyze the benefits for the US investor of diversifying internationally by combining his domestic ETF with ETFs of developed markets such as those of UK and Japan.

However, we also have to point out that numerous studies provide evidence of the attractiveness of emerging markets in international portfolio diversification motivated by their portfolio risk reduction due to their low correlation with developed markets (Errunza, 1977; Harvey, 1995; Koherts *et al.*, 1998; Christoffersen *et al.*, 2012). Consequently, we also analyze the benefits for the US investor of diversifying internationally combining his domestic ETF with ETFs of emerging markets such as Brazil and Malaysia (the Morgan Stanley Capital International iShares for Brazil and Malaysia respectively). Finally, we also provide the results obtained combining all these five ETFs of developed and emerging markets.

Furthermore, we compare our results with those obtained from the naïve rule and the classical sample-based mean-variance approach. This comparison allows us to contribute to the ongoing debate about the usefulness of a mean-variance allocation rule when it is compared to a naïve diversification providing results with a daily frequency and in an international context. Moreover, we also compare our results with those obtained from a multivariate GARCH approach in which returns are modeled on mean return level only. Note that this approach exclusively considers the dynamics in correlations and volatilities while the expected returns are the same as those obtained employing their sample counterparts. Consequently, it lets us to compute the additional economic value provided by considering the dynamics in cross-market returns with a VAR-GARCH approach.

² Previous papers, such as Serban *et al.* (2007) and García-Álvarez and Luger (2011), employ these ETFs with the simplest practical applications.

We evaluate these alternative strategies in terms of realized out-of-sample Sharpe ratios and use the bootstrap methodology proposed by Ledoit and Wolf (2008) in order to estimate the statistical significance of the differences between Sharpe ratios.

Our overall results show that it is possible to out-perform the naïve rule with an optimal international diversification strategy. Additionally, our results suggest that taking into account the cross-market return and correlation dynamics estimated from a multivariate VAR-GARCH model leads to diversification benefits. The economic value of this allocation approach persists even when the costs associated with the daily rebalance of each portfolio are considered. These findings are relevant not only for academics, but also for practitioners, especially professional portfolio managers.

The remainder of the paper is organized as follow. In section 2 we describe the methodology employed to construct and evaluate the proposed international diversification strategy. Section 3 describes the database employed. In section 4 we present the empirical results. Finally, in section 5 we provide the main conclusions.

2. Methodology

This section is divided into three main sub-sections. Firstly, we present the multivariate model employed to estimate conditional returns and volatilities for the ETFs. Secondly, we describe the methodology for the construction of the optimal portfolios. Finally, we describe the criterion employed to evaluate the performance of the alternative strategies.

2.1. The multivariate VAR-GARCH approach

The econometric specification used in this paper has two components. Firstly, a vector autorregresion (VAR) with k lags is used to model the returns. This allows for autocorrelations and cross-autocorrelations in the returns:

$$R_{i,t} = c_i + \sum_{k=1}^K \sum_{i=1}^N \alpha_{ij} R_{i,t-k} + \varepsilon_{i,t} \quad [1]$$

$$\varepsilon_{it} | \Omega_{t-1} \approx N(0, H_t) \quad [2]$$

where $R_{i,t}$ are the daily returns for the iShares, c_i and α_{ij} for $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, N$ are the parameters to be estimated, $\varepsilon_{i,t}$ indicates the innovations for each index at time t following a normal distribution with a mean of 0 and variance H_t where Ω_{t-1} is the information set in $t-1$.

Secondly, to model the second-order moments, we employ the Dynamic Conditional Correlation, DCC-GARCH approach introduced by Engle (2002) which is a generalization of the Bollerslev (1990) Constant Conditional Correlation (CCC) model.

In Bollerslev's model the covariance matrix H_t at time t is denoted as follows:

$$H_t = D_t R D_t \quad [3]$$

where $D_t = \text{diag}(\sqrt{h_{it}})$ is a $N \times N$ matrix containing the time varying conditional standard deviations obtained from an univariate GARCH model and R is a correlation matrix containing the conditional correlations of the standardized residuals (ε_t).

The DCC model differs from the CCC model in allowing R to be time varying:

$$H_t = D_t R_t D_t \quad [4]$$

R_t remains the correlation matrix but now is a time varying $N \times N$ correlation matrix with diagonal elements equal to 1 which is defined as follows:

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2} \quad [5]$$

where $Q_t = \{q_{ij,t}\}$ is a covariance matrix of the standardized residuals denoted as:

$$Q_t = (1 - \alpha - \beta) \bar{Q} + \alpha (\varepsilon_{t-1} \varepsilon'_{t-1}) + \beta Q_{t-1} \quad [6]$$

\bar{Q} denotes the unconditional correlation matrix of the standardized residuals, $\text{diag}(Q_t)^{-1/2}$ is a diagonal matrix containing the diagonal elements of the $N \times N$ positive definite matrix Q_t and α and β are non-negative scalar parameters such that $\alpha + \beta < 1$.

For a pair of assets i and j their conditional correlation at time t can be defined as:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} = \frac{(1 - \alpha - \beta)\bar{q}_{ij} + \alpha\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta q_{ij,t-1}}{\sqrt{[(1 - \alpha - \beta)\bar{q}_{ii} + \alpha\varepsilon_{i,t-1}^2 + \beta q_{ii,t-1}][1 - \alpha - \beta)\bar{q}_{jj} + \alpha\varepsilon_{j,t-1}^2 + \beta q_{jj,t-1}]}} \quad [7]$$

Under the Gaussian assumption, the log-likelihood of the estimators is:

$$L = -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log|D_t| + \log|R_t| + \varepsilon_t' R_t^{-1} \varepsilon_t) \quad [8]$$

Finally, we note that in this model the conditional variance-covariance matrix H_t is estimated in two steps. Firstly, a univariate GARCH model is estimated together with the conditional mean specification as in equation [1] for each of the ETFs. Secondly, the standardized residuals from the estimated GARCH are used subsequently in a simple specification to model and measure the time-varying correlation matrix.

2.2. Optimal portfolio choice

According to the classical theory of optimal portfolio selection by Markowitz (1952), managers will allocate wealth among the ETFs to minimize portfolio variance subject to the constraint that the expected portfolio return attains a specific target R^* . In our case this is the expected return of the naïve strategy, following the influential work of Kirby and Ostdiek (2012). In doing this, the problem which portfolio managers face is to find the optimal portfolio weights w_t which solve the optimization problem:

$$\begin{aligned} \min_{w_t} \quad & w_t' \mathbf{H}_{t+1|t} w_t \\ \text{s.t.} \quad & w_t' E\{\mathbf{R}_{t+1}\} \geq R^* \\ & w_t' \mathbf{1} = 1 \\ & w_t \geq 0 \end{aligned} \quad [9]$$

where $\mathbf{1}$ is a vector of ones and the non-negativity constraints $w_t \geq 0$ mean that the portfolio manager is prohibited from making short sales.

It should be pointed out that these constraints would be valid if mutual fund managers were the only players in the international investment context. However, hedge fund managers³ may participate as well. For that reason, we also consider a version of [9] without those short-sale constraints so that the optimal solution w_t may contain negative weights indicating short positions. In each case, the portfolio optimization problem is a standard quadratic programming problem that is readily solved numerically.

As we described above, the practical application of this programming exercise may differ depending on the methodology employed to estimate the expected return and the covariance matrix of the asset under consideration. The classical mean-variance approach, also called plug-in method, consists of computing the sample mean and covariance matrix of asset returns up to time t . However, we follow previous research and employ two model-based approaches. Firstly, we employ a GARCH-based approach which consists of using forward-looking volatility forecasts from a multivariate GARCH model. However, the implied return forecasts in this case are the same as that of the plug-in approach. Consequently, this technique only differs from the traditional mean-variance in its forecasts of the conditional covariance matrix. Finally, we employ an approach based on the use of one-day-ahead return and volatility forecasts from the proposed multivariate VAR-GARCH model. This allows us to directly compute the additional economic value provided by considering the dynamics in cross-market returns in the optimization exercise.

2.3. Performance evaluation

We consider the out-of-sample Sharpe ratio as the measure of portfolio performance. It is defined as the sample mean of out-of-sample excess returns over the risk-free asset,⁴ divided by their sample standard deviation:

³ Note that only authorized participants, such as retail and institutional investors, actually buy or sell shares of an ETF directly.

⁴ We employ the US Treasury Bill rate as the risk-free interest rate, obtained from Kenneth R. French's website.

$$SR_p = \frac{\hat{\mu}_p}{\hat{\sigma}_p} \quad [10]$$

As reported by García-Álvarez and Luger (2011), it is necessary to consider this measure to evaluate portfolio performance because it is the most ubiquitous risk-adjusted measure used by financial market practitioners to rank fund managers and to evaluate the attractiveness of investment strategies in general.

Furthermore, to test whether our optimal portfolios produce economically significant profits, we calculate the portfolio performances after taking into account the costs associated with the daily rebalance of each portfolio considering not only transaction costs but also daily portfolio turnovers.

Following DeMiguel *et al.* (2009), we denote the share of wealth in area i before the portfolio is rebalanced at time $t+1$ as:

$$\hat{\omega}_{i,t^+} = \frac{\hat{\omega}_{i,t} (1 + R_{i,t+1})}{\sum_{i=1}^N \hat{\omega}_{i,t} (1 + R_{i,t+1})} \quad [11]$$

When the portfolio is rebalanced it gives rise to a trade in area i of magnitude $|\hat{\omega}_{i,t+1} - \hat{\omega}_{i,t^+}|$, where $\hat{\omega}_{i,t+1}$ is the optimal portfolio weight on area i at time $t+1$ after rebalancing. Consequently, the total amount of turnover across all assets in the portfolio is:

$$\tau_{t+1} = \sum_{i=1}^N |\hat{\omega}_{i,t+1} - \hat{\omega}_{i,t^+}| \quad [12]$$

Moreover, if c denotes the proportional transactions cost, then the total cost to rebalance the portfolio is $c \times \tau_{t+1}$. Let $R_{p,t+1} = \sum_{i=1}^N R_{i,t+1} \hat{\omega}_{i,t}$ denote the portfolio return from a given strategy before rebalancing occurs. The evolution of wealth invested according to that strategy is then given by:

$$W_{t+1} = W_t (1 + R_{p,t+1}) (1 - c \times \tau_{t+1}) \quad [13]$$

and the simple return net of rebalancing costs is $R_{p,t+1}^c = W_{t+1}/W_t - 1$. Since the portfolio w_t is formed using only information available at time t and held for one day before being rebalanced at time $t+1$, the return $R_{p,t+1}^c$ represents the one-day out-of-sample return.

Finally, to assess the statistical significance of the differences between the performance of the benchmark strategy and those of the sample-based and model-based strategies, we employ a bootstrap inference method following Ledoit and Wolf (2008).

3. Database

For our empirical research we employ five ETFs which track the market indexes of the US, UK, Japan, Brazil and Malaysia. These financial instruments have been available since the early nineties, but they have grown increasingly popular in the last ten years. This is because they combine the range of an index portfolio with the simplicity of trading a single stock with lower administrative costs and lower capital gain taxes. Moreover, ETFs based on major indexes typically trade at much higher volumes than individual stocks enabling investors to get into and out of investment positions with minimum risk and expense.

More precisely, we have selected the Standard and Poor's Depository Receipt (SPDR) ETF which tracks the S&P500 as well as the Morgan Stanley Capital International iShares for the United Kingdom (MSCI UK), Japan (MSCI JP), Brazil (MSCI BZ) and Malaysia (MSCI MY) which track their respective MSCI stock market indexes. The motivation behind our selection is two-fold. Firstly, the SPDR is the most popular and actively trade ETF which tracks a major US stock market index. Secondly, the impact of the MSCI brand in the context of ETF industry allows their products to be the benchmarks for international portfolio managers.

These five ETFs are actively traded on the New York Stock Exchange during market hours. It enables us to rebalance our portfolios on a daily basis while avoiding the problem of non-synchronicity of the markets. Moreover, the data consists of daily returns (calculated as

logarithmic differences) for these five ETFs for the period from December 31, 2003 to December 31, 2013.⁵ To avoid in-sample overfitting as well as spurious findings,⁶ we use two non-overlapping sub-samples for the estimation and allocation stages: the first sub-sample (from December 31, 2003 to December 31, 2011 – 2,015 observations) is used for the estimation of the model, while the second sub-sample (from January 1, 2012 to December 31, 2013 - 502 observations) is used for the out-of-sample research.

Table 1

Table 1 presents the summary statistics for these return series over the estimation period. An initial conclusion might suggest that the performance of the SPDR as measured by mean daily return is better (0.014%) than the MSCI UK (0.011%) and the MSCI Japan (0.001%) but worse than the MSCI Brazil (0.070%) and the MSCI Malaysia (0.047%). However, on the basis of the Anova test we cannot reject the null hypothesis that all series in the group have the same mean since those differences are not statistically significant. On the other hand, due to the rejection of the null hypothesis of equality of variances among the return series, we can conclude that the SPDR is less volatile (standard deviation of 1.38%) than the other ETFs (1.74%, 1.56%, 2.69% and 1.48% for MSCI UK, MSCI Japan, MSCI Brazil and MSCI Malaysia respectively). Further tests produce similar results for these three series. Skewness and kurtosis values indicate that the distributions of returns for all the ETFs are skewed and leptokurtic. The Jarque-Bera statistic rejects the null hypothesis that the returns are normally distributed for all cases. The Ljung-Box statistic for up to 20 lags indicates the presence of significant linear and non-linear dependencies in the returns of all indexes. The ARCH test reveals that returns exhibit conditional heteroskedasticity and the augmented Dickey and Fuller and Philips and Perron tests indicate that these time series are stationary. On the basis of the features observed in Table 1, a VAR-GARCH approach is appropriate to estimate the first and second moments of these indexes.

⁵ We use logarithmic returns multiplied by 100 to facilitate the convergence of the empirical models.

⁶ Data snooping occurs if the same sample is used for both estimation and allocation.

4. Empirical results

This section is divided in two main sub-sections. The first one presents the in-sample estimates from the proposed multivariate VAR-GARCH models. The second one focuses on the out-of-sample application for international diversification purposes.

4.1. The multivariate VAR-GARCH models

The first step in the multivariate VAR-GARCH estimation procedure is to identify the best-fitting specification of the return series. This is particularly important as mis-specifying the mean equation may lead to an incorrect estimation of the variance equation (Ewing and Malik, 2005). Therefore, the conditional mean equations are defined as a VAR (k) process in which the k lag has been chosen following the Akaike information criterion. Once the mean structure is identified, we estimate the mean and variance specifications for each ETF while, in a further step, time-varying correlations are jointly estimated.

Tables 2 to 4

Tables 2 to 4 display the estimates of the multivariate VAR-GARCH model for each international diversification scenario by combining the SPDR, MSCI UK and MSCI Japan ETFs (Table 2), the SPDR, MSCI Brazil and MSCI Malaysia ETFs (Table 3) and all those five ETFs (Table 4). In Panel A of these tables we present the estimated coefficients of the VAR model and we observe the existence of significant dynamic relationships in the return series of these ETFs. In Panel B we observe that the coefficients for the lagged variance and shock-squared terms in the variance equations are highly significant for the three ETFs. This is consistent with time-varying volatility and confirms the appropriateness of the GARCH specification. In Panel C we note that the coefficients α and β are both statistically significant at conventional levels, supporting the presence of dynamic correlations in the markets. In addition, the sum of the coefficients is less than unity, implying that the correlation dynamics are mean reverting. Finally, for the purpose of robustness, we analyze the properties of the standardized residuals for each return series. Panel D reports the main results of these specification tests. As we can observe, in all cases the mean value is close to zero with a standard deviation of nearly one. We also note a reduction in the kurtosis of the residuals compared to the original series. The Ljung-Box Q statistics for the 20th orders performed over

the standardized residuals reveal a lack of serial autocorrelation and indicate the appropriate specification of the mean equations. Moreover, the Ljung-Box Q statistics for the 20th order in squared standardized residuals show that there is no series dependence in the squared standardized residuals, indicating the appropriateness of the fitted variance-covariance equations for the multivariate GARCH model.

Figure 1

Figure 1 presents the evolution of the estimated correlation coefficients obtained from the multivariate VAR-GARCH model for the most complex international diversification scenario in which we combine the five ETFs and shows that they vary considerably over short periods of time. However, if we examine the overall picture we see an upward trend, especially for the US-UK pair.⁷ These findings from the short periods of time are especially important for the purpose of our research because tactical asset allocation is only relevant if conditional correlations fluctuate over time (Goeij and Marquering, 2004) and this happens when we employ daily frequency data.

4.2. Out-of-sample results

We consider the problems faced by an active portfolio manager who rebalances his portfolios on a daily basis. To that end, with the parameters of the estimated VAR-GARCH models we forecast the return and the variance matrix of the ETFs considered in each investment scenario for the next trading day. We repeat this procedure 502 times (from January 1, 2012 until December 31, 2013) using a rolling sample of 2,015 observations in which the VAR-GARCH model is re-estimated each time. These forecasts are then used to determine the daily optimal weights for each index in the international portfolios proposed.

Nevertheless, the main objective of this section is to provide an out-of-sample performance evaluation of the portfolios we have constructed in terms of realized Sharpe ratios. To that end, and following DeMiguel *et al.* (2009) and Kirby and Ostdiek (2012), we consider the naïve rule as our benchmark strategy for comparison purposes, although we also provide results obtained using a sample-based as well as multivariate GARCH-based approaches.

⁷ These results are similar to those obtained by Christoffersen *et al.* (2012).

Tables 5 to 7

Tables 5 to 7 show the performance evaluation results obtained for each investment scenario. Panel A of each table provides the results for the benchmark strategy while Panels B and C provide the results for the optimal portfolios with and without short-sale constraints respectively in order to examine whether the imposition of short-sale constraints lead to improved portfolio performances (Jagannathan and Ma, 2003). Within these panels we present the results for the sample-based, GARCH-based and VAR-GARCH-based approaches. In all cases the reported out-of-sample results include: summary statistics for each portfolio in terms of annualized returns and annualized standard deviation; the annualized Sharpe ratios and the resulting two-sided bootstrap p -values for tests of the equality of the Sharpe ratio with those of the benchmark strategy obtained using the methodology suggested in Ledoit and Wolf (2008); and the portfolio turnover, defined as the average sum of the absolute value of the trades across the three available indexes, in order to get a sense of the amount of trading required to implement each portfolio strategy. Finally, each table provides the results when there are no transaction costs and assuming transactions costs of 50 basic points as in DeMiguel *et al.* (2009).

We observe in Tables 5 and 6 that the sample-based and model-based strategies significantly out-perform the naïve strategy. Not only are larger returns obtained through the mean-variance portfolios but they also provide smaller standard deviations. Consequently, they yield higher Sharpe ratios than the naïve rule. Moreover, we observe that the portfolio with higher results in terms of the risk-return trade-off is that based on a VAR-GARCH approach. These results persist even when short-sale is allowed and when the costs associated with the daily rebalance of each portfolio are taken into account and they are not dependent on whether the selected ETFs track the most relevant indexes from developed or emerging markets.

However, in Table 7 we present the results obtained when we combine five instead of three ETFs of developed as well as emerging markets and we observe that the effect of short sale restrictions are quite important in this scenario. The sample-based and model-based strategies significantly out-perform the naïve strategy. However, the VAR-GARCH approach only

reports the best Sharpe ratio when short-sale is prohibited. These results are in line with the findings of Jagannathan and Ma (2003) who show that imposing a short-sale constraint amounts to shrinking the extreme elements of the covariance matrix and thereby stabilizes the portfolio weights.

Figure 2

Finally, Figure 2 displays the cumulative returns of the proposed portfolios after considering daily rebalancing costs over the out-of-sample period in each investment scenario. Again we note the better performance of the optimal portfolio based on the return and volatility forecast from a multivariate VAR-GARCH approach which produces positive and upward cumulative returns far higher than the naïve rule.

Our overall results indicate that active portfolio managers can construct optimal portfolios in an international context and beat their common benchmarks. However, our results also reveal that it is necessary to include the prediction of expected returns in the quadratic programming problem in order to design valuable portfolio strategies.

6. Conclusions

Over the last decade, international diversification has become less attractive as the correlation among markets has steadily risen. This is due to the increased interconnectedness of different economies as well as the globalization of financial markets which has intensified due to the current crisis. This paper contributes to the ongoing debate providing an alternative approach for constructing optimal portfolios with out-of-sample economic gains.

To that end, we employ ETFs instead of market indexes. This allows us to construct optimal portfolios on a daily basis without non-synchronous problems as well as to model time series with a multivariate VAR-GARCH model. Consequently, our strategy is based on the use of time-varying return and volatility forecasts from the multivariate approach for the most active exchange trade funds of the US, UK, Japan, Brazil and Malaysia over the last decade.

Our main results confirm that it is possible to out-perform the naïve diversification strategy and classical active portfolio techniques when we construct optimal portfolios which include an accurate prediction of expected returns in the optimization problem. These findings are relevant for academics and for active professional managers who can use this technique to add value to their strategies in the current context.

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Table 1: Descriptive Statistics

	SPDR	MSCI UK	MSCI JP	MSCI BZ	MSCI MY
Mean	0.014	0.011	0.001	0.070	0.047
Median	0.081	0.070	0.000	0.191	0.085
Std. Dev.	1.382	1.743	1.566	2.698	1.487
Skewness	-0.058	-0.243	0.159	-0.350	-0.377
Kurtosis	15.889	14.030	12.291	11.013	9.024
Jarque-Bera	13949.4 (0.00)	10234.1 (0.00)	7256.5 (0.00)	5431.6 (0.00)	3094.5 (0.00)
Q (20)	92.448 (0.00)	90.327 (0.00)	102.57 (0.00)	75.403 (0.00)	94.119 (0.00)
Q ² (20)	2559.4 (0.00)	2536.5 (0.00)	2569.4 (0.00)	2920.2 (0.00)	674.9 (0.00)
ARCH(20)	711.10 (0.00)	644.88 (0.00)	693.35 (0.00)	814.61 (0.00)	260.21 (0.00)
ADF(4)	-21.960 (0.00)	-21.849 (0.00)	-21.495 (0.00)	-21.499 (0.00)	-21.499 (0.00)
PP(6)	-49.794 (0.00)	-51.234 (0.00)	-51.837 (0.00)	-45.967 (0.00)	-45.967 (0.00)

This table presents descriptive statistics for the daily return series of the ETFs for US, UK, Japan, Brazil and Malaysia over the in-sample period (2004-2011). Skewness and Kurtosis refer to the series skewness and kurtosis coefficients. The Jarque-Bera statistic tests the normality of the series. This statistic has an asymptotic $\chi^2(2)$ distribution under the normal distribution hypothesis. Q(20) and Q²(20) are Ljung-Box tests for 20th-order serial correlation in the returns and squared returns. ARCH (20) is the Engle (1982) test for the 20th-order ARCH. These three tests are distributed as $\chi^2(20)$. The ADF (4) and PP (6) refer to the Augmented Dickey and Fuller (1981) and Phillips and Perron (1988) unit root tests corresponding to the process with intercept but without trend. The *p*-values of these tests are reported in parenthesis.

Table 2: Estimates from the VAR – GARCH model. Scenario 1

	SPDR		MSCI UK		MSCI JP	
<i>Panel A: VAR parameters</i>						
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
c_i	0.059***	(0.00)	0.049*	(0.06)	0.035	(0.18)
$R_{US,t-1}$	-0.034	(0.39)	0.088*	(0.08)	-0.078*	(0.08)
$R_{US,t-2}$	-0.084*	(0.06)	0.010	(0.84)	0.001	(0.98)
$R_{UK,t-1}$	-0.005	(0.86)	-0.136***	(0.00)	0.015	(0.69)
$R_{UK,t-2}$	0.035	(0.23)	0.014	(0.72)	0.022	(0.58)
$R_{JP,t-1}$	-0.024	(0.28)	-0.011	(0.70)	-0.071**	(0.04)
$R_{JP,t-2}$	0.006	(0.81)	-0.035	(0.22)	-0.023	(0.48)
<i>Panel B: GARCH parameters</i>						
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
c_i	0.017**	(0.03)	0.025**	(0.02)	0.051***	(0.00)
$\varepsilon_{i,t-1}^2$	0.090***	(0.00)	0.090***	(0.00)	0.096***	(0.00)
$h_{ii,t-1}$	0.898***	(0.00)	0.902***	(0.00)	0.880***	(0.00)
<i>Panel C: DCC persistence parameters</i>						
	Coeff.	p-value				
α	0.030***	(0.00)				
β	0.961***	(0.00)				
<i>Panel D: Residual diagnosis</i>						
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Mean	-0.039		-0.021		-0.026	
Std. Dev.	1.000		1.001		1.000	
Skewness	-0.593		-0.557		-0.374	
Kurtosis	5.062		5.181		4.662	
JB	474.29***	(0.00)	502.77***	(0.00)	278.54***	(0.00)
Q (20)	14.489	(0.81)	13.328	(0.86)	17.027	(0.65)
Q ² (20)	24.116	(0.24)	25.141	(0.20)	15.026	(0.78)

This table reports the estimations from the multivariate VAR-GARCH model by combining the SPDR, MSCI UK and MSCI Japan ETFs. Panel A shows the mean equation estimations for the ETF return series (*p*-values in parenthesis). Panel B shows the variance equation estimations with *p*-values in parenthesis. Panel C shows the DCC persistence parameters. Panel D reports summary statistics for the standardized residuals. Q (20) stands for the Ljung-Box Q statistic for the standardized residuals up to 20 lags while Q² (20) stands for the Ljung-Box Q statistic for the squared standardized residuals up to 20 lags (*p*-values reported in parenthesis)

***, ** and * represent the levels of significance of 1%, 5% and 10% respectively.

Table 3: Estimates from the VAR – GARCH model. Scenario 2

	SPDR		MSCI BZ		MSCI MY	
<i>Panel A: VAR parameters</i>						
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
c_i	0.062***	(0.00)	0.135***	(0.00)	0.075**	(0.01)
$R_{US,t-1}$	-0.055	(0.11)	-0.133*	(0.09)	-0.124***	(0.00)
$R_{US,t-2}$	-0.036	(0.30)	0.010	(0.90)	-0.065	(0.15)
$R_{US,t-3}$	-0.047	(0.17)	-0.112	(0.11)	-0.077*	(0.07)
$R_{BZ,t-1}$	0.016	(0.24)	0.080**	(0.03)	0.042**	(0.03)
$R_{BZ,t-2}$	0.006	(0.60)	-0.024	(0.46)	0.027	(0.16)
$R_{BZ,t-3}$	-0.005	(0.69)	-0.011	(0.73)	0.027	(0.11)
$R_{MY,t-1}$	-0.040*	(0.07)	-0.069	(0.20)	-0.104**	(0.01)
$R_{MY,t-2}$	-0.023	(0.23)	-0.032	(0.54)	0.007	(0.82)
$R_{MY,t-3}$	0.035	(0.13)	-0.067	(0.18)	0.032	(0.31)
<i>Panel B: GARCH parameters</i>						
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
c_i	0.016**	(0.03)	0.160***	(0.00)	0.013*	(0.06)
$\varepsilon_{i,t-1}^2$	0.092***	(0.00)	0.094***	(0.00)	0.059***	(0.00)
$h_{ii,t-1}$	0.897***	(0.00)	0.878***	(0.00)	0.938***	(0.00)
<i>Panel C: DCC persistence parameters</i>						
	Coeff.	p-value				
α	0.034***	(0.00)				
β	0.958***	(0.00)				
<i>Panel D: Residual diagnosis</i>						
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Mean	-0.041		-0.030		-0.023	
Std. Dev.	0.999		1.000		0.999	
Skewness	-0.604		-0.439		-0.722	
Kurtosis	5.061		4.086		8.420	
JB	477.41***	(0.00)	163.34***	(0.00)	2633.9***	(0.00)
Q (20)	15.542	(0.75)	25.378	(0.19)	17.453	(0.62)
Q ² (20)	24.008	(0.24)	18.265	(0.57)	5.302	(0.99)

This table reports the estimations from the multivariate VAR-GARCH model by combining the SPDR, MSCI Brazil and MSCI Malaysia ETFs. Panel A shows the mean equation estimations for the ETF return series (p -values in parenthesis). Panel B shows the variance equation estimations with p -values in parenthesis. Panel C shows the DCC persistence parameters. Panel D reports summary statistics for the standardized residuals. Q (20) stands for the Ljung-Box Q statistic for the standardized residuals up to 20 lags while Q² (20) stands for the Ljung-Box Q statistic for the squared standardized residuals up to 20 lags (p -values reported in parenthesis). ***, ** and * represent the levels of significance of 1%, 5% and 10% respectively.

Table 4: Estimates from the VAR – GARCH model. Scenario 3

	SPDR		MSCI UK		MSCI JP		MSCI BZ		MSCI MY	
<i>Panel A: VAR parameters</i>										
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
c_i	0.061***	(0.00)	0.047*	(0.08)	0.034	(0.20)	0.139***	(0.00)	0.072***	(0.01)
$R_{US,t-1}$	-0.043	(0.31)	0.011	(0.84)	-0.122**	(0.06)	-0.094	(0.38)	-0.106*	(0.06)
$R_{US,t-2}$	-0.067*	(0.09)	-0.019	(0.73)	0.015	(0.79)	-0.009	(0.93)	-0.115*	(0.05)
$R_{US,t-3}$	-0.037	(0.39)	-0.052	(0.34)	-0.081	(0.16)	-0.154	(0.11)	-0.095	(0.11)
$R_{US,t-4}$	0.049	(0.26)	-0.033	(0.55)	0.001	(0.99)	-0.035	(0.72)	-0.022	(0.68)
$R_{UK,t-1}$	0.001	(0.99)	-0.164***	(0.00)	-0.001	(0.97)	0.010	(0.89)	0.007	(0.86)
$R_{UK,t-2}$	0.037	(0.23)	-0.005	(0.91)	0.022	(0.58)	0.012	(0.87)	0.034	(0.41)
$R_{UK,t-3}$	-0.001	(0.98)	-0.015	(0.73)	0.005	(0.89)	0.066	(0.35)	0.043	(0.31)
$R_{UK,t-4}$	0.001	(0.99)	-0.032	(0.45)	0.013	(0.73)	0.109	(0.12)	0.033	(0.39)
$R_{JP,t-1}$	-0.024	(0.29)	-0.036	(0.26)	-0.076***	(0.03)	-0.074	(0.20)	-0.045	(0.14)
$R_{JP,t-2}$	0.006	(0.81)	-0.039	(0.19)	-0.027	(0.43)	0.008	(0.89)	0.033	(0.31)
$R_{JP,t-3}$	-0.004	(0.86)	0.028	(0.32)	-0.030	(0.39)	-0.014	(0.79)	-0.006	(0.84)
$R_{JP,t-4}$	-0.038*	(0.08)	-0.038	(0.17)	-0.076***	(0.02)	-0.100*	(0.06)	-0.031	(0.33)
$R_{BZ,t-1}$	0.020	(0.14)	0.061***	(0.00)	0.052***	(0.01)	0.084**	(0.02)	0.049***	(0.01)
$R_{BZ,t-2}$	0.001	(0.98)	0.031**	(0.06)	-0.005	(0.81)	-0.025	(0.47)	0.018	(0.37)
$R_{BZ,t-3}$	-0.007	(0.59)	0.010	(0.55)	0.038**	(0.06)	-0.015	(0.64)	0.018	(0.30)
$R_{BZ,t-4}$	-0.010	(0.44)	-0.004	(0.83)	0.015	(0.43)	-0.040	(0.26)	0.030*	(0.08)
$R_{MY,t-1}$	-0.036	(0.11)	0.031	(0.38)	-0.019	(0.59)	-0.065	(0.23)	-0.099***	(0.01)
$R_{MY,t-2}$	-0.028	(0.16)	-0.008	(0.77)	-0.012	(0.68)	-0.036	(0.50)	-0.003	(0.92)
$R_{MY,t-3}$	0.033	(0.19)	-0.016	(0.59)	-0.014	(0.66)	0.052	(0.31)	0.023	(0.46)
$R_{MY,t-4}$	0.007	(0.77)	0.002	(0.95)	0.037	(0.32)	0.007	(0.89)	-0.016	(0.61)

Table 4: Estimates from the VAR – GARCH model. Scenario 3 (Cont.)

	SPDR		MSCI UK		MSCI JP		MSCI BZ		MSCI MY	
<i>Panel B: GARCH parameters</i>										
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
c_i	0.016**	(0.03)	0.023**	(0.03)	0.046***	(0.00)	0.154***	(0.00)	0.012*	(0.03)
$\varepsilon_{i,t-1}^2$	0.092***	(0.00)	0.092***	(0.00)	0.096***	(0.00)	0.093***	(0.00)	0.060***	(0.00)
$h_{ii,t-1}$	0.897***	(0.00)	0.900***	(0.00)	0.882***	(0.00)	0.881***	(0.00)	0.937***	(0.00)
<i>Panel C: DCC persistence parameters</i>										
	Coeff.	p-value								
α	0.050***	(0.00)								
β	0.900***	(0.00)								
<i>Panel D: DCC Residual diagnosis</i>										
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Mean	-0.040		-0.021		-0.027		-0.029		-0.020	
Std. Dev.	0.999		1.000		1.000		1.000		1.001	
Skewness	-0.595		-0.556		-0.342		-0.452		-0.701	
Kurtosis	5.086		5.052		4.368		4.067		8.342	
JB	482.54***	(0.00)	455.90***	(0.00)	195.73***	(0.00)	163.72***	(0.00)	2553.12***	(0.00)
Q (20)	14.398	(0.81)	12.953	(0.88)	17.763	(0.61)	23.673	(0.26)	16.255	(0.70)
Q ² (20)	22.622	(0.31)	24.261	(0.23)	16.454	(0.69)	18.174	(0.58)	5.278	(0.99)

This table reports the estimations from the multivariate VAR-GARCH model by combining the SPDR, MSCI UK, MSCI Japan, MSCI Brazil and MSCI Malaysia ETFs. Panel A shows the mean equation estimations for the ETF return series (p -values in parenthesis). Panel B shows the variance equation estimations with p -values in parenthesis. Panel C shows the DCC persistence parameters. Panel D reports summary statistics for the standardized residuals. Q (20) stands for the Ljung-Box Q statistic for the standardized residuals up to 20 lags while Q² (20) stands for the Ljung-Box Q statistic for the squared standardized residuals up to 20 lags (p -values reported in parenthesis).

***, ** and * represent the levels of significance of 1%, 5% and 10% respectively.

Figure 1: Conditional correlations

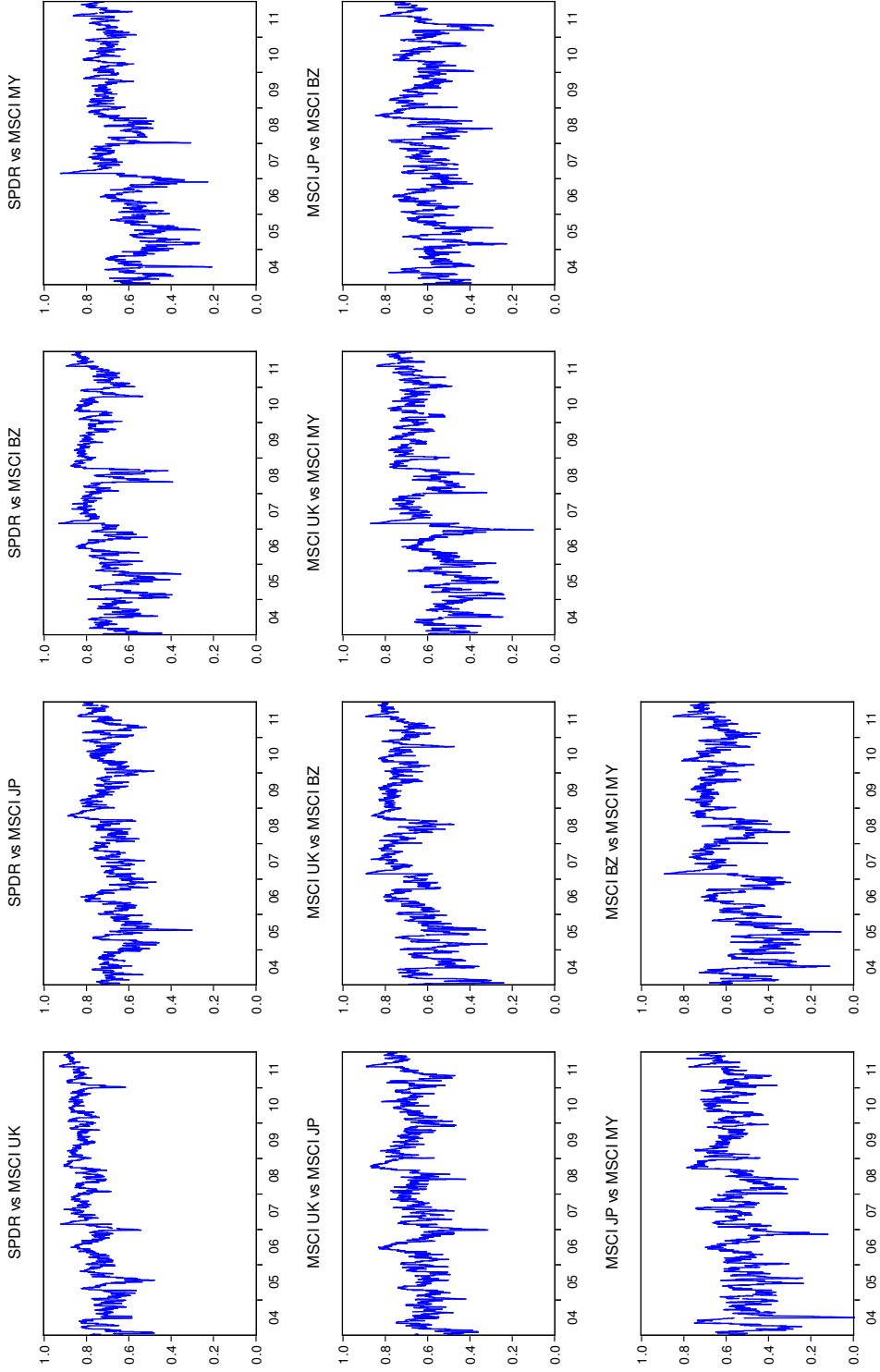


Table 5: Out-of-sample performance evaluation. Scenario 1.

	Before transaction costs			After transaction costs			
	Return	St.Dev.	SR (p-val)	Turnover	Return	St.Dev.	SR (p-val)
<i>Panel A: Benchmark strategy</i>							
Naive	17.880	13.214	1.349	2.199	13.781	13.096	1.048
<i>Panel B: Optimal portfolios with short-sale constraints</i>							
Sample	20.149	12.294	1.634 (0.00)	1.548	17.062	12.176	1.396 (0.00)
GARCH	18.820	11.973	1.567 (0.00)	0.772	16.928	11.851	1.423 (0.00)
VAR-GARCH	20.663	12.091	1.704 (0.00)	0.909	18.485	11.959	1.541 (0.00)
<i>Panel C: Optimal portfolios without short-sale constraints</i>							
Sample	21.977	12.549	1.749 (0.00)	4.161	15.598	12.350	1.258 (0.00)
GARCH	18.871	11.652	1.614 (0.00)	1.544	16.236	11.531	1.403 (0.00)
VAR-GARCH	25.610	12.686	2.014 (0.00)	1.917	22.064	12.433	1.770 (0.00)

This table reports the out-of-sample performance evaluation of the proposed portfolios before and after transaction costs based on the annualized mean, the annualized standard deviation, the mean turnover and the annualized Sharpe ratios with the resulting bootstrap p -values reported in parenthesis obtained using the methodology suggested in Ledoit and Wolf (2008). Panel A presents the results for the naive rule which serves as our benchmark strategy for comparison purposes. Panel B presents the results from the mean-variance portfolio strategies with short sales constraints and Panel C without short sale constraints.

Table 6: Out-of-sample performance evaluation. Scenario 2.

	Before transaction costs			After transaction costs			
	Return	St.Dev.	SR (p-val)	Turnover	Return	St.Dev.	SR (p-val)
<i>Panel A: Benchmark strategy</i>							
Naive	7.582	14.299	0.526	1.723	4.370	14.090	0.306
<i>Panel B: Optimal portfolios with short-sale constraints</i>							
Sample	13.511	13.035	1.032 (0.00)	1.098	11.550	12.913	0.890 (0.00)
GARCH	17.135	11.729	1.456 (0.00)	0.859	15.330	11.608	1.315 (0.00)
VAR-GARCH	19.824	12.649	1.562 (0.00)	1.124	17.324	12.503	1.381 (0.00)
<i>Panel C: Optimal portfolios without short-sale constraints</i>							
Sample	17.629	13.500	1.301 (0.00)	1.973	13.601	13.341	1.015 (0.00)
GARCH	21.910	11.382	1.920 (0.00)	11.546	11.180	16.055	0.693 (0.00)
VAR-GARCH	25.848	13.582	1.899 (0.00)	2.701	21.280	13.282	1.598 (0.00)

This table reports the out-of-sample performance evaluation of the proposed portfolios before and after transaction costs based on the annualized mean, the annualized standard deviation, the mean turnover and the annualized Sharpe ratios with the resulting bootstrap p -values reported in parenthesis obtained using the methodology suggested in Ledoit and Wolf (2008). Panel A presents the results for the naive rule which serves as our benchmark strategy for comparison purposes. Panel B presents the results from the mean-variance portfolio strategies with short sales constraints and Panel C without short sale constraints.

Table 7: Out-of-sample performance evaluation. Scenario 3.

	Before transaction costs			After transaction costs			
	Return	St.Dev.	SR (p-val)	Turnover	Return	St.Dev.	SR (p-val)
<i>Panel A: Benchmark strategy</i>							
Naive	10.978	13.673	0.799	1.637	7.841	13.486	0.577
<i>Panel B: Optimal portfolios with short-sale constraints</i>							
Sample	16.545	12.238	1.347 (0.00)	1.195	14.240	12.114	1.170 (0.00)
GARCH	15.662	11.844	1.317 (0.00)	2.348	13.002	11.794	1.097 (0.00)
VAR-GARCH	18.518	12.091	1.527 (0.00)	1.284	15.933	11.948	1.329 (0.00)
<i>Panel C: Optimal portfolios without short-sale constraints</i>							
Sample	25.132	12.138	2.066 (0.00)	25.135	7.460	19.976	0.370 (0.00)
GARCH	20.114	11.356	1.766 (0.00)	2.109	16.934	11.197	1.507 (0.00)
VAR-GARCH	19.268	11.083	1.733 (0.00)	2.618	15.622	10.900	1.428 (0.00)

This table reports the out-of-sample performance evaluation of the proposed portfolios before and after transaction costs based on the annualized mean, the annualized standard deviation, the mean turnover and the annualized Sharpe ratios with the resulting bootstrap p -values reported in parenthesis obtained using the methodology suggested in Ledoit and Wolf (2008). Panel A presents the results for the naive rule which serves as our benchmark strategy for comparison purposes. Panel B presents the results from the mean-variance portfolio strategies with short sales constraints and Panel C without short sale constraints.

Figure 2: Cumulative returns

