

# Hedge funds and asset markets: tail or two-state dependence? \*

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## Abstract

This paper tries to reconcile opposing evidence found in previous literature about the tail neutrality of Market Neutral Hedge Funds. In particular, we show that the existence of a common regime between hedge funds and the market index creates a non-linear dependence that can be confounded with tail dependence. This evidence is consistent with the Market Neutral Hedge Funds be affected by a common macroeconomic component as opposed to holding a portfolio subject to big losses in extremely rare events. We estimate a regime-switching copula model and we show with simulated data from our model that the tests conducted in [Brown and Spitzer \(2006\)](#) do not reject the tail dependence hypothesis, even if the tail dependence is set to zero. Moreover, we provide evidence of statistical and economically significant linear correlations in each regime.

*JEL classification:* G11, G23.

*Keywords:* Hedge funds, market neutrality, regime-switching models, copula, tail dependence.

## 1 Introduction

It is widely thought that hedge funds offer an advantage over other investment vehicles in that they are immune from day-to-day market fluctuations, which makes them attractive to wealthy individuals and institutional investors, especially during uncertain times.

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Indeed, numerous empirical studies have found low correlations between hedge fund returns and market returns (see for example, [Agarwal and Naik \(2004\)](#) and [Fung and Hsieh \(1999\)](#)). This characteristic has propelled growth in an industry whose size in 2013 was estimated to have been \$2.63 trillion.<sup>1</sup>

The growth of this seemingly opaque industry has fuelled research on the risk exposures of hedge funds to asset markets.<sup>2</sup> Knowledge of this is particularly important, as crashes in this industry might lead to potentially devastating effects in financial markets. For instance, hedge funds have been implicated in the 1992 crisis that led to major exchange realignments in the European Monetary System (EMU), the 1994 crisis in bond markets, and the 1997 East Asian financial crisis. And of course, the most widely known event involving hedge funds was the near-collapse of Long Term Capital Management (LTCM) in 1998. While hedge fund activities might not cause financial crises,<sup>3</sup> they may, however, pose a potential threat that may amplify its effects.

Hedge funds are usually classified by their investment styles. One such investment style is called market neutral hedge funds (MNHF), which refer to “funds that actively seek to avoid major risk factors, but take bets on relative price movements utilising strategies such as long-short equity, stock index arbitrage, convertible bond arbitrage, and fixed income arbitrage” ( [Fung and Hsieh \(1999\)](#), p. 319). As [Fung and Hsieh \(2001\)](#) and [Patton \(2009\)](#) note, MNHFs are not only one of the largest, but also are among the fastest-growing investment styles in the industry.

Recent literature has investigated the “neutrality” of MNHFs to the market index, of which there are numerous definitions. The most prominent one, which is the focus

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<sup>1</sup>Source: “ETFs/ETPs Grew At A Faster Rate Than Hedge Funds In 2013”, [nasdaq.com](#), February 18, 2014.

<sup>2</sup>For recent studies, see [Billio et al. \(2009\)](#) and [Kelly and Jiang \(2012\)](#).

<sup>3</sup>[Ben-David et al. \(2012\)](#) document that hedge funds in the US withdrew their equity holdings during the 2007-2009 financial crisis.

of numerous empirical studies, is the exposure of these funds to tail risk.<sup>4</sup> While they have found that there is low correlation between MNHFs and the market index, there is no consensus whether this particular style of hedge funds is exposed to tail risk or not. [Brown and Spitzer \(2006\)](#) propose a tail neutrality measure which uses a simple binomial test for independence, and find that hedge funds exhibit tail dependence. They also compare their measure with results from logit regressions similar to the study by [Boyson et al. \(2010\)](#), and find that while both are able to capture tail dependence, their tail neutrality measure performs considerably better. [Patton \(2009\)](#), meanwhile, proposes a test statistic using results from extreme value theory and finds that there is no tail dependence between MNHFs and the market index. The analyses performed in the previous papers are developed in a static framework.<sup>5</sup> [Distaso et al. \(2010\)](#), meanwhile, use hedge fund index data to model the dependence using a time-varying copula, and find that there does not exist tail dependence between hedge fund index returns and market index returns.

A common thread of all these papers, however, is that the results hinge on the assumption that the joint distribution of hedge fund and asset market returns is fairly static over time. This paper, in turn, departs from the literature by allowing for regime switches, both in the joint distribution of hedge fund index and asset market returns, and their corresponding marginal distributions. To operationalise this, we model the joint distribution of market-neutral hedge fund returns and asset market returns, which we represent by the market index return, through a regime-switching Student- $t$  copula proposed by [da Silva Filho et al. \(2012\)](#).<sup>6</sup> Meanwhile, we model the marginal distribution

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<sup>4</sup>[Patton \(2009\)](#) examines four other neutrality concepts: “mean neutrality”, “variance neutrality”, “Value-at-Risk neutrality” and “complete neutrality”, which we do not explicitly address in this paper.

<sup>5</sup>[Patton and Ramadorai \(2013\)](#) use a dynamic framework to analyse risk exposures of hedge funds to different asset classes. However, they do not explicitly study tail dependence.

<sup>6</sup>[Rodriguez \(2007\)](#) uses a regime-switching copula to analyse contagion between different asset markets in Asia during the 1997 financial crisis. [da Silva Filho et al. \(2012\)](#), meanwhile, studies stock market dependence of the US, UK, and Brazilian stock market indices.

as an asymmetric Student- $t$  distribution proposed by Galbraith and Zhu (2010)). We depart from da Silva Filho et al. (2012) by allowing the parameters of the marginal distributions to depend on the regime.

Our contribution to the literature is threefold. First, we find evidence that the shifts in the marginal distributions of MNHF's and the market follow a common regime; this shift creates a non-linear dependence that cannot be captured by models with smooth dynamics in the dependence nor by a model as the one proposed by da Silva Filho et al. (2012) where the copula parameters are regime-dependent, but the parameters of the marginal distributions are not.<sup>7</sup>

Second, as in most of the previous literature, we find evidence of small correlation between the MNHF and the Market; however, we find that there exists an economically and statistically significant correlation conditioning on the regime. In particular, we find a negative correlation in the bust regime which is consistent with the MNHF cutting positions in response to the market declines as found in Patton and Ramadorai (2013) and Ben-David et al. (2012), and a positive correlation in normal periods.

Third, we reconcile previous evidence on tail dependence by showing that tests that reject the null of no unconditional tail dependence using MNHF data (Brown and Spitzer (2006)), also reject the null hypothesis with simulated data from our model without tail dependence. As these tests are based on the sample tails, if the sample is not extremely large they cannot differentiate between the non-linear dependence from the regime switches and tail dependence. However, when the tests are based on extrapolation such as copula methods or extreme value theory, they will not reject the null hypothesis.

The rest of the paper is as follows. Section 2 reviews the definition of tail dependence.

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<sup>7</sup>There exist a copula (Sklar theorem) which will capture the whole dependence in the unconditional distribution, however in this paragraph we stressed that this dependence cannot be capture with copulas commonly used to measure tail dependence

Section 3 discusses the data used in the estimations. The modelling approach used in this paper, and the estimation results, are discussed in Section 4. Section 5 relates the current model with the [Brown and Spitzer \(2006\)](#) study that finds tail dependence between MNHFs and asset markets. Section 6 concludes.

## 2 Tail dependence and copulas

Dependencies between (extreme) financial asset returns have gained increasing attention from academics and market practitioners in recent times, particularly after the global financial crisis of 2007-2009.<sup>8</sup> If financial assets are positively correlated in periods of turmoil, particularly those in the extremes, then it might lead to a negative impact on the asset portfolios of large institutional investors, which might weaken their financial stability. By this logic, investigating the dependence of hedge funds, which are generally thought to be uncorrelated with the market, and asset markets makes it all the more important.

In practice, the concept of tail dependence has been used to measure not only extreme financial asset dependence, but also non-linear dependence. We provide the definition of tail dependence below.

**Definition:** Given a bivariate random vector  $(X, Y)$  with marginal c.d.f.  $F_X$  and  $F_Y$ , we say that  $X$  has lower tail dependence with respect to  $Y$  if and only if

$$\lambda_L \equiv \lim_{u \rightarrow 0^+} \text{Prob}(X < F_X^{-1}(u) | Y < F_Y^{-1}(u)) > 0$$

where  $\lambda_L$  is defined as the lower tail dependence coefficient.<sup>9</sup>

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<sup>8</sup>Previous empirical literature such as [Karolyi and Stulz \(1996\)](#) and [Longin and Solnik \(2001\)](#) have documented an increase in dependencies between individual stock and market index returns in international financial markets during periods of market stress.

<sup>9</sup>The upper tail dependence coefficient is given by:  $\lambda_U \equiv \lim_{u \rightarrow 1^-} \text{Prob}(X > F_X^{-1}(u) | Y > F_Y^{-1}(u))$ .

One way of estimating tail dependence is through copula methods, which are frequently used in practice. An advantage of doing this is that copulas, by construction, allow the characterisation of the dependence structure of the joint distribution of financial asset returns independently of their marginal distributions. In particular, given a bivariate random vector  $(X, Y)$  with marginal c.d.f.'s  $(F_X, F_Y)$  and copula function  $C(F_X(x), F_Y(y))$ , the lower tail dependence is given by:

$$\lambda_L = \lim_{u \rightarrow 0^+} \frac{C(u, u)}{u} \quad (1)$$

In this vein, we estimate the tail dependence between MNHFs and the asset market through a regime-switching copula, which uses the whole data set in order to identify the parameters of the copula function. Through the specific parametric form of the copula, we then take the limit and obtain the tail dependence measure.<sup>10</sup> We also consider another broad class of estimation procedures used in the literature, which we label as sample tail-based estimation. These techniques drop most of the sample and focus only on the relationship between the  $p\%$  lowest values in the sample. An example of this type of technique are the logit regressions considered in Section 5. In this paper, we test how accurate the inference with sample tail-based techniques is when the true data generating process (DGP) has some non-linear dependence different from tail dependence, and in particular, a Markov-switching common factor.

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<sup>10</sup>We refer to this estimation technique and similar ones as interpolation techniques, as the entire history from the available data is used to recover the lower tail dependence measure.

### 3 Data

The dataset we use for the study is obtained from three sources. We collect daily hedge fund return data from the Hedge Fund Research (HFR) database from April 1, 2003 to November 8, 2013. We only consider hedge fund indices that are of the “equity market neutral” type, which is indicated in the substrategy type of the database. Although we have data for global indices, we cannot clearly identify from the database the type of the hedge fund, we only work with data from the United States. To represent the simple daily market index return, we obtain NYSE Composite data from Datastream. We finally obtain data on recession dates from the National Bureau of Economic Research (NBER).

Table 1 presents summary statistics of the hedge fund index and the NYSE Composite. We observe that for non-crisis periods, both MNHFs and NYSE indices have positive mean return, have less dispersion, and fatter tails. During crisis periods, meanwhile, we find that the mean return is negative, have more dispersion, and thinner tails. We find, however, that the hedge fund index is negatively skewed during both crisis and non-crisis periods, while the market index is positively skewed during crisis periods, and negatively skewed during non-crisis periods. Unconditionally, however, we find that the hedge fund returns have negative mean return, while the market index has positive mean return.

Table 2, meanwhile presents dependence statistics between the hedge fund index and the market. We present three dependence statistics: the Pearson correlation coefficient, the Kendall- $\tau$  coefficient, and the left tail dependence statistic implied by a Clayton copula. We find that during NBER crisis periods, the market index and the hedge fund index exhibit negative correlation. During NBER non-crisis periods, however, we find that the market index and the hedge fund index exhibit positive correlation. The

Pearson correlation coefficient and the Kendall- $\tau$  differ, however, when we take into account unconditional dependence. The Pearson correlation coefficient shows that there does not exist any dependence, while the Kendall- $\tau$  exhibits positive dependence. These statistics indicate that there does not exist unconditional correlation but some non-linear dependence between the MNHF and the Market that could explain a positive Kendall  $\tau$ . This fact motivates the use of a regime switching model that could capture non-linear dependence.<sup>11</sup>

## 4 Tail or two-state dependence?

In this section, we discuss the model specification for the empirical analysis we pursue, and the subsequent results.

### 4.1 Model specification

In order to analyse the dependence between MNHFs and the market index, we appeal to the model proposed by [Rodriguez \(2007\)](#), which extends Patton's (2006) conditional copula model by introducing a hidden Markov chain to capture unobserved regime-switching. More formally, let  $\{(x_{Ft}, x_{Mt})\}_{t=1}^T$ ,  $t = 1, \dots, T$  be the MNHF and market returns, respectively. To model the dependence between these two variables, we represent the joint distribution through a regime-switching copula as follows:

$$F(x_{Ft}, x_{Mt}|s_t, \mathcal{I}_{t-1}) = C_{\theta_{ct}}(F_F(x_{Ft}|s_t, \mathcal{I}_{t-1}), F_M(x_{Mt}|s_t, \mathcal{I}_{t-1})|s_t, \mathcal{I}_{t-1}) \quad (2)$$

where  $C_{\theta_{ct}}$  is the conditional copula with time-varying parameters  $\theta_{c,s_t}$ ,  $s_t$  is the state,  $\mathcal{I}_{t-1}$  is the set of all possible information, and  $F_i(x_{it}|s_t, \mathcal{I}_{t-1})$  ( $i = M, F$ ) are the marginal

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<sup>11</sup>Interestingly, though, we do not observe any dependence from the left tail parameter of the Clayton copula.

c.d.f's of  $x_{it}$ .

The marginal distributions are assumed to be:

$$x_{it} = \mu_{s_t} + \rho_{s_t}(x_{it-1} - \mu_{s_t}) + \sigma_{s_t}\varepsilon_{it} \quad (3)$$

where  $\varepsilon_{it} \sim f_{AST}(\varepsilon; \alpha_{s_t}, \nu_{1,s_t}, \nu_{2,s_t}, \mu_{s_t}, \sigma_{s_t})$ . We assume that  $\varepsilon_{it}$  follows an asymmetric student- $t$  distribution proposed by Galbraith and Zhu (2010), an extension of the two-piece method by Hansen (1994), that allows for an additional skewness parameter. This distributional assumption has two main advantages: First, it is extremely flexible and, second, it has a c.d.f. that can be efficiently computed which reduces the estimation time since we need to compute the quantile of each observation and time period. The density has the following form:

$$f_{AST}(x; \theta) = \begin{cases} \frac{\alpha}{\alpha^*} K(\nu_1) \left[ 1 + \frac{1}{\nu_1} \left( \frac{x}{2\alpha^*} \right)^2 \right]^{-\frac{\nu_1+1}{2}}, & x \leq 0 \\ \frac{1-\alpha}{1-\alpha^*} K(\nu_2) \left[ 1 + \frac{1}{\nu_2} \left( \frac{x}{2(1-\alpha^*)} \right)^2 \right]^{-\frac{\nu_2+1}{2}}, & x > 0 \end{cases} \quad (4)$$

where  $\theta = (\alpha, \nu_1, \nu_2)^T$ ,  $\alpha \in (0, 1)$  is the skewness parameter,  $\nu_1 > 0$ ,  $\nu_2 > 0$ , are the left and right tail parameters respectively,  $K(\nu) \equiv \Gamma((\nu+1)/2)/[\sqrt{\pi\nu}\Gamma(\nu/2)]$ , and  $\alpha^*$  is defined as:

$$\alpha^* = \alpha K(\nu_1)/[\alpha K(\nu_1) + (1-\alpha)K(\nu_2)] \quad (5)$$

Denote  $\mu$  as the location parameter, and  $\sigma$  as the scale parameter, the general form of this density, which is what we estimate, is:  $\frac{1}{\sigma} f_{AST}\left(\frac{x-\mu}{\sigma}; \theta\right)$ .

The copula model chosen is the student- $t$  copula, which has the following parame-

terisation:

$$\begin{aligned}
C_{\theta_{ct}}(u_1, u_2 | s_t, \mathcal{I}_{t-1}) &= \int_{-\infty}^{\tau_{\delta_{s_t}}^{-1}(u_1)} \int_{-\infty}^{\tau_{\delta_{s_t}}^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\delta_{s_t}^2}} \\
&\times \left(1 + \frac{r^2 - 2\delta_{s_t}rs + s^2}{\eta_{s_t}(1-\delta_{s_t}^2)}\right)^{-\frac{\eta_{s_t}+1}{2}} dr ds
\end{aligned} \tag{6}$$

where the copula parameters  $\delta_{s_t}$ , which is the correlation (dependence) parameter, and  $\eta_{s_t}$ , which is the degrees of freedom parameter, are allowed to change with the state. This parameterisation provides the following advantages. First, it allows the series to be negatively correlated. Second, it allows for different degrees of tail dependence depending on the state. However, it has some limitations since by construction upper and lower tail dependence are equal and non-negative.<sup>12</sup>

The hidden regime is modeled as a first-order Markov chain with two different states that we labeled *Boom* and *Bust*.<sup>13</sup>

$$P(s_t = \textit{Boom} | s_{t-1} = \textit{Boom}, s_{t-2}, \dots, s_0) = P(s_t = \textit{Boom} | s_{t-1} = \textit{Boom}) = p_{11} \quad \forall t$$

We estimate the parameters of the model via maximum likelihood estimation (MLE), wherein we use the Hamilton filter outlined in [Hamilton \(1989\)](#) to facilitate the process. Standard errors are obtained using the Hessian matrix.

## 4.2 Results

We discuss the results of the estimation in this subsection. Tables 3 and 4 present the estimation results for the marginal distributions. We focus first on Table 3, which shows

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<sup>12</sup>As can be observed in Figure 3 where the empirical joint density is plotted, this assumptions do not seem far fetched.

<sup>13</sup>We also considered three states and the results remain the same; in this case, the third endogenous state is very close to the *Boom* state.

results for the marginal distribution of the hedge fund indices. We find that during *boom* periods, the marginal distributions of hedge funds have a positive location and scale shift. The skewness parameter is also positive. During *bust* periods, hedge funds have negative location shift and positive scale shift parameters, and a positive skewness parameter, though it is less than that of the *boom* period. Meanwhile, from Table 4 it can be observed that the market index return exhibits positive location and scale shift, and skewness parameters both during *boom* and *bust* periods. Table 5 presents the moments implied by the regime-switching model, which were obtained through simulation. We find that during *boom* periods, both MNHFs and market index exhibit a positive mean return, while during *bust* periods, they exhibit negative return, albeit small in magnitude. The marginal distributions are less dispersed in *boom* periods than in *bust* periods. Moreover, the marginal distributions during *bust* periods exhibit fatter tails than those in *boom* periods. However, marginal distributions of MNHFs exhibit positive skewness during *boom* periods and negative skewness in *bust* periods, which is not the case for stock returns, where the signs are opposite.

Table 6 presents results from the estimated conditional copulas. Focusing on the dependence parameter, we find that hedge funds and asset markets exhibit positive correlation during *boom* periods, and negative correlation during *bust* periods. The tail dependence parameter, however, is significantly positive, both during the *boom* and the *bust* periods. These results confirm the hypothesis drawn from the summary statistics that MNHFs do not exhibit economically significant tail dependence but that their marginal distribution shifts with the states in the same direction as that of the market. Additionally, the model finds a high positive correlation in the *boom* periods and a low negative correlation in the *bust* periods. The positive linear correlation is an interesting feature of the data that seems to be robust to different specifications, and it might be driven because of demand reasons (investors prefers zero returns and a low correlation

than zero correlation and lower returns in "good" times) and/or by supply reasons (during *boom* periods, it might be more costly or even impossible to hedge some market risks).<sup>14</sup>

Finally, Figures 1 and Figure 2 present the smoothed probabilities, and the joint distribution, respectively. We find that the smoothed probabilities closely approximate the actual occurrence of the *boom* and *bust* periods during the periods specified, including the European turmoil. Figure 2 presents the joint distribution of the data and our fitted model by their deciles. We observe that the model is able to capture most of the dependence structure; moreover, we find that fat tails resemble to tail dependence in the 10 % and 90 % decile. Additionally, the negative correlation and the flexibility of our distribution are able to capture some of the negative dependence in the 10% and 90% percentile.<sup>15</sup>

## 5 Reconciliation with previous literature

The insight gained from the previous section is that hedge funds and asset returns do not exhibit economically significant tail dependence, which is in line with most of the previous literature. Instead, we find that hedge funds and asset markets have a non-linear dependence that is dependent on a common regime. In this section, meanwhile, we test if the methodology used in [Brown and Spitzer \(2006\)](#) to reject the hypothesis of no tail dependence is accurate when the true DGP corresponds to the regime-switching model we have earlier outlined. In their paper, they use monthly individual hedge fund data and illustrate the dependence with the market using rank-rank plots; similarly, we simulate a similar sample of 70 hedge funds with 37 months of data from our model, and

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<sup>14</sup>[Billio et al. \(2009\)](#) argue that a (latent) idiosyncratic risk factor exposure that was common to all hedge funds during both the 1998 LTCM/Russian bonds crisis and the 2008 global financial crisis.

<sup>15</sup>We use negative dependence to refer to the fact that  $Prob(x < q_{x,10\%} | y < q_{y,90\%}) > 0.1$ , where  $q_{z,p}$  is defined by  $Prob(z < q) = p$ .

construct the rank-rank plot. Figure 3 compares the rank-rank plot obtained from the model, with that obtained by [Brown and Spitzer \(2006\)](#). In these plots, each bin has a color that ranges from cyan, which implies that the bin is (nearly) empty, to magenta, which implies that this bin has most of the observations. As expected, the rank-rank plots look closely similar, with the model being able to replicate the observation that the lower tails contain most of the observations.

[Brown and Spitzer \(2006\)](#) also propose two tail neutrality tests based on the binomial test and a logit regression. We discuss each test in turn. The first test divides the distribution into 4 quadrants, which we illustrate in Figure 4:

1. LL for observations where the market and the hedge fund are both below the median;
2. LW for observations where the fund is below the median, but the market is not;
3. WL for observations where the market is below the median, but the hedge fund is not; and
4. WW for observations where the neither the market nor the hedge fund are below the median.

[Brown and Spitzer \(2006\)](#) calculate the standard odds ratio, which is provided by the following formula:

$$Odds = \frac{LL \times WW}{LW \times WL} \quad (7)$$

As implied by the formula, there exists tail dependence if the ratio is greater than 1, which implies that the LL and WW quadrants have more observations than the LW and WL quadrants. This test, however, assumes a centred and stable distribution over the whole sample which it is not consistent with the empirical results presented in the previous sections. To test if the non-linear dependence implied by the regime-switching

model is accepted by the binomial test as evidence of tail dependence, we simulate 10,000 samples of a bivariate time series with a length of 2,600 days (similar to our sample and [Brown and Spitzer \(2006\)](#) sample) from our estimated model without tail dependence, where we change the tail dependence parameter to 0. For each time series, we calculate the odds ratio, and compare the resulting statistic with the  $p$ -value implied by independence, and a bivariate Normal distribution.<sup>16</sup> We also calculate results from a chi-squared test of independence, which [Brown and Spitzer \(2006\)](#) note as a stronger test of tail dependence. We finally calculate the proportion of simulated data that amounts to a rejection of the null hypothesis of tail dependence. Table 7 presents results from the binomial tests for independence. We find that we are able to reject the null hypothesis of no tail dependence for all of the simulations performed. These results are robust to the sample size and the number of simulations.

The second test, which is based from [Boyson et al. \(2010\)](#), is a logit regression of a dummy that takes value 1 if the hedge fund return is below its  $p$  percentile, on the market return and the same dummy for the market index:

$$\Pr(x_F < q_{F,p}) = \Lambda(\mu + \rho_1 x_M + \rho_0 \mathbb{1}\{x_M < q_{M,p}\}) \quad (8)$$

where  $x_F$  and  $x_M$  are the hedge fund and market returns respectively,  $q_{Z,p}$  is defined by:  $\Pr(x_Z < q_{Z,p}) = p$ ,  $Z = \{M, F\}$ ;  $\Lambda(\cdot)$  is the logit c.d.f and  $\mathbb{1}\{\cdot\}$  is the indicator function. If there exists tail dependence the coefficient of the market dummy should be positive. The main issue in this test is the selection of the percentile  $p$ . In particular, [Brown and Spitzer \(2006\)](#) use 15%. We do several regressions using different values of  $p$  from 20% to 0.1%. We perform these regressions for each of the simulated time series,

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<sup>16</sup>Although [Brown and Spitzer \(2006\)](#) suggest a continuity correction as inference is based on the log odds ratio with a standard error equal to  $\sqrt{\frac{1}{LL} + \frac{1}{LW} + \frac{1}{WL} + \frac{1}{WW}}$ , we do not do it here as the simulations always result in a sufficient number of observations for each bin.

and calculate, for each tail cut-off, the proportion of rejection of the null hypothesis of no tail dependence.

Figure 5 presents the results of the logit regressions. We find that when the tail cut-off is large, almost all of the simulations have significant coefficients, which [Brown and Spitzer \(2006\)](#) take as evidence of tail dependence. However, as the tail cut-off becomes smaller, we find that the rate by which we can reject the null hypothesis of no tail dependence becomes smaller. In fact, looking at the extreme tails, we find that the rejection rate whittles to 5% of the simulations. This result suggests that indeed, there is no tail dependence between the market index and hedge fund returns.

We consider this empirical evidence as a reconciliation between two opposing results in the literature. On one hand, studies using interpolation techniques like copula methods to compute tail dependence measures find there was no evidence of such; on the other hand, studies using the sample tails to make inference consistently reject the hypothesis of no tail dependence. With simulated data from our model, we show that indeed, if the hedge fund and the market follow a two-state Markov-switching model with no tail dependence, we are able to conclude that there exists tail dependence between the hedge fund and the market.

## 6 Conclusions

A distinct feature that separates hedge funds from other financial assets is its seemingly low correlation with financial markets. Previous empirical literature has investigated the “neutrality” of market neutral hedge funds with the market index; in particular, the literature has addressed the question of whether there exists tail dependence between hedge funds and financial markets. However, there does not seem to be any agreement in the literature as to the relationship between hedge funds and financial markets.

This paper revisits this question by employing a regime-switching copula model. Results from our estimation indicate that conditional on the state of the economy, MNHFs and market returns exhibit either positive or negative dependence. Moreover, the results suggest that there exists a (latent) regime-dependent factor that drives hedge fund returns.

We then reconcile the results we obtain with previous literature by conducting the tests in the previous literature by simulating data from the regime-switching model we have earlier proposed. We find that, indeed, the model is able to generate the tail dependence observed in previous studies. These results imply that by not taking into account regime-switching dependence, we might inaccurately conclude that hedge funds and asset markets indeed exhibit positive left tail dependence.

An interesting avenue for future research is to investigate which risk factors are important during *boom* and *bust* periods. While previous literature has found that hedge funds are indeed exposed to systematic risk factors (see [Agarwal and Naik \(2004\)](#) and [Patton \(2009\)](#) for examples), taking into account which factors are important in different regimes allows for a more precise estimation of risk modelling. This, in turn, allows for a more comprehensive picture on the diversification benefits of hedge funds.

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Table 1: Summary statistics of hedge funds and the NYSE

	Unconditional	NBER Crisis	Non-Crisis
	Market Neutral Fund		
Mean	-0.002	-0.012	0.000
Std	0.261	0.372	0.238
Skewness	-0.107	-0.198	-0.011
Kurtosis	20.135	4.326	29.644
	NYSE		
Mean	0.000	-0.001	0.001
Std	0.013	0.026	0.010
Skewness	-0.177	0.082	-0.351
Kurtosis	13.868	6.128	6.812

Note: The table describes summary statistics of market neutral hedge fund index returns and the NYSE Composite index return. These are observed on a daily frequency from April 2003 to November 2013.

Table 2: Dependence statistics between the market and the hedge fund index

	Unconditional	NBER Crisis	Non-Crisis
Pearson	0.0147 (0.4481)	-0.2344 (0.0000)	0.1811 (0.0000)
Kendall	0.0501 (0.0001)	-0.1558 (0.0000)	0.1015 (0.0000)
Left Tail Dep. (Clayton Copula)	0.0000 (1.0000)	0.0079 (0.9845)	0.0000 (1.0000)

Note: The table describes dependence statistics between market neutral hedge fund (MNHF) index returns and the NYSE Composite index return. We compute three dependence statistics: the Pearson correlation coefficient, the Kendall- $\tau$  dependence statistic, and the left tail dependence parameter computed from a symmetrised Clayton-Joe copula. Standard errors are in parentheses.

Table 3: Estimates of the marginal distribution of the hedge fund index

	Bust		Boom	
$\alpha$ (skewness)	0.452	(0.037)	0.570	(0.026)
$\nu_1$ (left tail)	3.598	(0.835)	24.654	(21.167)
$\nu_2$ (right tail)	5.205	(1.357)	4.121	(0.790)
$\mu$ (location)	-0.067	(0.035)	0.041	(0.013)
$\rho$ (autocorrelation)	0.063	(0.033)	0.004	(0.024)
$\sigma$ (scale)	0.268	(0.012)	0.164	(0.005)

Note: The table presents the parameters of the marginal distribution of market index returns, which is assumed to be an asymmetric student-t proposed by [Galbraith and Zhu \(2010\)](#). Standard errors are in parentheses, and were computed using the numerical Hessian. Estimates are from NYSE Composite market index data from April 2003 to November 2013.

Table 4: Estimates of the marginal distribution of the market index

	Bust		Boom	
$\alpha$ (skewness)	0.572	(0.035)	0.552	(0.026)
$\nu_1$ (left tail)	3.964	(0.933)	6.921	(2.062)
$\nu_2$ (right tail)	3.096	(0.689)	5.704	(1.459)
$\mu$ (location)	0.003	(0.002)	0.002	(0.001)
$\rho$ (autocorrelation)	-0.092	(0.033)	-0.061	(0.022)
$\sigma$ (scale)	0.015	(0.001)	0.006	(0.000)

Note: The table presents the parameters of the marginal distribution of market index returns, which is assumed to be an asymmetric student-t proposed by [Galbraith and Zhu \(2010\)](#). Standard errors are in parentheses, and were computed using the numerical Hessian. Estimates are from NYSE Composite market index data from April 2003 to November 2013.

Table 5: Moments implied by the model

	Unconditional	<i>Bust</i>	<i>Boom</i>
	(a.) Market Neutral Fund		
Mean	-0.001	-0.023	0.009
Std	0.261	0.368	0.194
Skewness	-0.282	-0.371	0.5444
Kurtosis	22.098	16.296	9.118
	(b.) NYSE Composite		
Mean	0.000	-0.001	0.001
Std	0.014	0.023	0.008
Skewness	0.3454	0.4128	-0.195
Kurtosis	97.409	47.414	5.544

Note: The table presents the moments implied by the estimated model. Moments are obtained through simulation.

Table 6: Estimates of the conditional copula parameters

	Bust		Boom	
$p_{jj}$	0.990	(0.004)	0.996	(0.002)
$\delta$	-0.048	(0.042)	0.218	(0.026)
$\eta$	7.524	(2.580)	14.908	(6.720)
Tail Dep.	0.014	(0.000)	0.006	(0.000)
Kendall $\tau$	-0.031	(0.001)	0.140	(0.000)

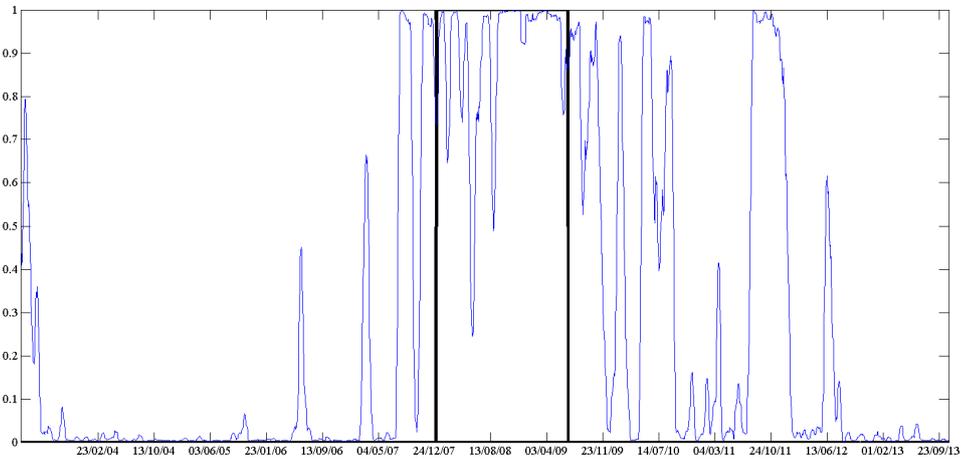
Note: The table presents the parameters of the conditional copula of hedge fund and market index returns, which is assumed to be an student-t copula proposed by [da Silva Filho et al. \(2012\)](#). Standard errors are in parentheses, and were computed using the numerical Hessian. Estimates are from HFR and NYSE Composite market index data from April 2003 to November 2013.

Table 7: Binomial test results

	$N=1,000$	$N=10,000$
Chi-squared test	1.000	1.000
Odds ratio test		
<i>Independence</i>	1.000	1.000
<i>bivariate Normal</i>	1.000	1.000

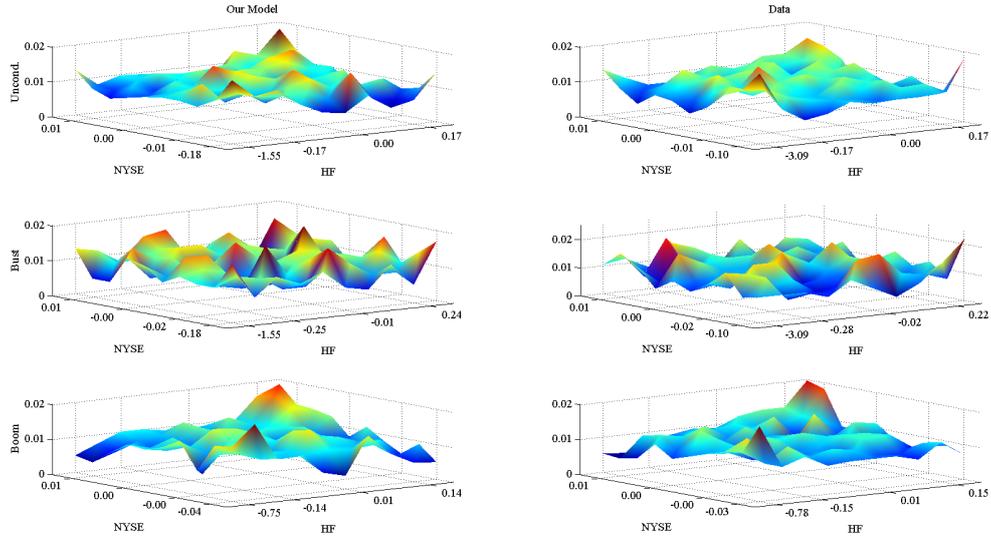
Note: The table presents proportions of simulated data from the regime-switching model proposed in section 3 that reject the null hypothesis of no tail dependence using the tail dependence measures proposed by [Brown and Spitzer \(2006\)](#).

Figure 1: Smoothed Probabilities



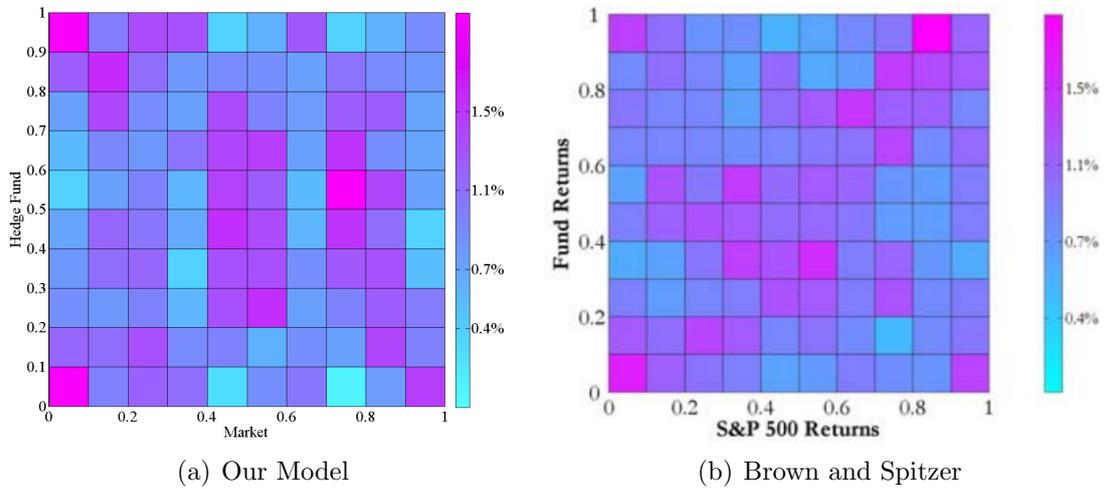
The solid line presents the 10-days moving average smoothed probability of being in the state labeled as *Bust*. The region inside the vertical lines correspond to the NBER recession period.

Figure 2: Joint distribution



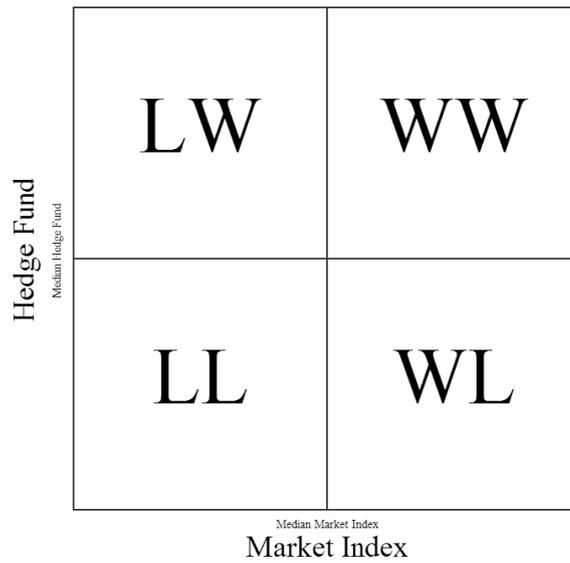
Note: This figure compares one simulation from our model with the empirical joint p.d.f from the data. We classify as *Boom* (*Bust*) periods those days whose probability of being in the *Boom* state is higher (lower) than 0.5.

Figure 3: Rank-Rank plot



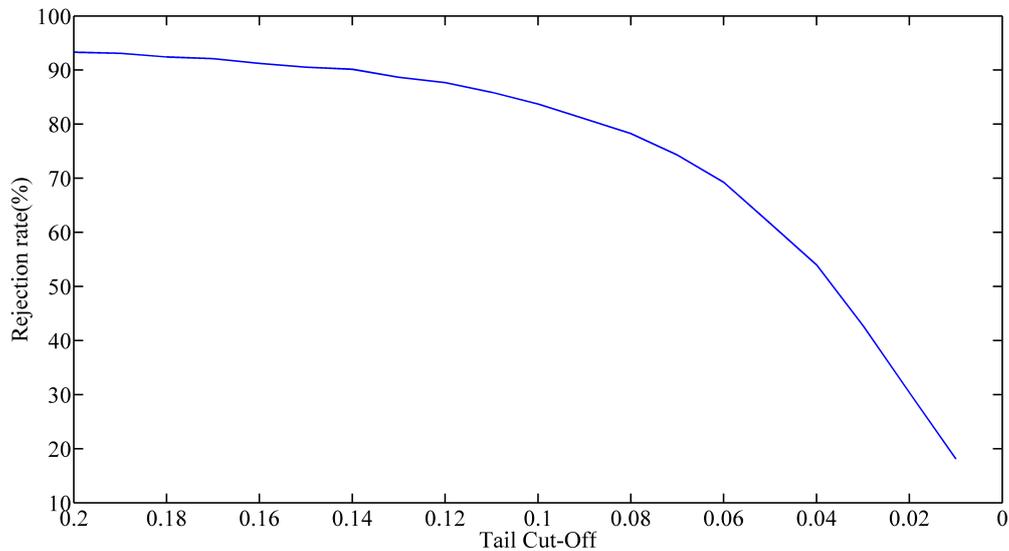
Note: This figure compares the rank-rank plot for the hedge fund and market joint distribution computed with data from our simulated model with the rank-rank plot in Brown and Spitzer (2006). The rank-rank plot is constructed by dividing each of the marginals in deciles and cross tabulate the two distribution deciles, thus a magenta square in (0,0)-(0.1,0.1) means that hedge fund returns below the 10 % decile tend to coincide with market returns below the 10% decile.

Figure 4: Binomial test Illustration



Note: This diagram presents the division of the joint distribution in order to compute the binomial test.

Figure 5: Logit test results



Note: This figure presents the actual size of the test with the null hypothesis  $H_0 : rho_0 = 0$  in the following model:  $Prob(x_F < q_{F,p}) = \Lambda(\mu + \rho_1 x_M + \rho_0 \mathbb{1}\{x_M < q_{M,p}\})$  where  $x_F$  and  $x_M$  are the hedge fund and market returns respectively,  $q_{Z,p}$  is defined by:  $Prov(x_Z < q_{Z,p}) = p$   $Z = \{M, F\}$ ,  $\Lambda(\cdot)$  is the logit c.d.f and  $\mathbb{1}\{\cdot\}$  is the indicator function. In the vertical axis is presented the rejection rate and in the horizontal axis are presented the different values of  $p$ .