Modelling Electricity Swaps with Stochastic Forward Premium Models

Iván Blanco[a], Juan Ignacio Peña[b] and Rosa Rodríguez[c]

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Abstract

In this paper we develop a stochastic forward premium model for the pricing and hedging of electricity derivatives. Besides a few general factors affecting the whole swap curve within each market segment (yearly, quarterly and monthly contracts), each point of the curve is exposed to unique risk factors. The general factors are (i) the average swap prices and (ii) deterministic seasonal factors and the unique factors are the stochastic forward premiums associated with each point in the swap curve. The general and unique stochastic factors are driven by processes which follow the Multivariate Normal Inverse Gaussian (MNIG) distribution, which allow for stochastic dynamics in terms of correlated Lévy processes. We estimate the model with data from the European Energy Exchange (EEX) using an extensive panel data set of swap prices. The model captures the basic stylized facts and in particular the volatilities, correlations, asymmetries and kurtosis. The specific component in each swap contract represents a non-negligible source of risk and cannot be hedged by using other swap contracts. Our model fits the data better than models based on spot prices, or models based on the Heath-Jarrow-Morton approach. Value-at-Risk measures based on spot price models and HJM models strongly underestimate tail risk, the extent of underestimation varies across the segments of the swap market. On the other hand, the stochastic forward premium model estimates accurately the actual tail risk.

Keywords: Electricity swaps, Stochastic forward premium, Multivariate Normal Inverse Gaussian distribution, Lévy processes

JEL Codes: C51; G13; L94; Q40

[a] Universidad Carlos III de Madrid, Department of Business Administration, c/ Madrid 126, 28903 Getafe (Madrid, Spain). ivan.blanco@uc3m.es;[b] Corresponding author. Universidad Carlos III de Madrid, Department of Business Administration, c/ Madrid 126, 28903 Getafe (Madrid, Spain). ypenya@eco.uc3m.es; [c] Universidad Carlos III de Madrid, Department of Business Administration, c/ Madrid 126, 28903 Getafe (Madrid, Spain). rosa.rodriguez@uc3m.es.Juan Ignacio Peña and Rosa Rodriguez acknowledge financial support from the Ministry of Economics and Competitiveness, respectively, through grant ECO2012-35023 and thorough grant ECO2012-36559. We thank Alvaro Cartea and other participants in the Energy Finance 2014 conference for their useful suggestions as well as to Diego Fresoli for his suggestions on Matlab.

1. Introduction

A growing number of electricity market participants use derivatives contracts as a way of trading synthetic electricity generation plants[1]. Therefore electricity derivatives are becoming progressively an important part of the global energy commodities market. By far, the most liquid derivatives contracts in the electricity markets are forwards, futures and swaps[2].

Models for pricing and hedging financial derivatives on energy prices can be classified in three broad categories: (i) based on fundamental equilibrium (ii) based on spot energy prices and other key variables such as convenience yields or interest rates and (iii) based on forward price processes. Models in the first category focus on supply and demand relationships to obtain the power prices as a solution of an optimization problem. This optimization problem embodies information on market prices and trading activity. This allows the computation of the forward prices, using the condition that they provide equilibrium in the demand for forward contracts; see Bessembinder and Lemmon (2002). In a similar vein, Supatgiat, Zhang and Birge, (2001) show that market clearing prices are determined by solving a Nash equilibrium problem for the bidding strategies of market agents. Although useful for a wide range of applications, models in this category do not capture appropriately the price dynamics, which is what market participants need in order to develop effective hedging and risk management strategies.

The second category is based on specifying stochastic processes of the spot price and possibly of a limited set of other state variables, calibrating their parameters using market data. Then, one may resort to closed formulas or numerical approximations, in order to price contingent claims. Examples of this approach are Schwartz (1997), Hilliard and Reis (1998), Schwartz and Smith (2000), and Casassus and Collin-

[1]For instance in Europe the EFET (www.efet.org) a group of more than 100 energy trading companies from 27 European countries promotes energy trading throughout Europe and provides templates of many standardized energy derivatives contracts.

[2] In most electricity markets, forward and futures contracts guarantee delivery of the electricity over a period of time (e.g. monthly or yearly contracts) rather than at a fixed future time. As Benth and Koekebakker (2008) argue the nature of these contracts are very similar to a swap exchanging a fixed price for floating (spot) electricity price during the defined period of time. In fact swap contracts are integrals of traditional fixed delivery time forward contracts. Therefore we will call futures or forwards with delivery over a given period swaps.

Dufresne (2005), among others. This approach has been successfully applied to some energy commodities, particularly crude oil, but its adequacy is less clear in the case of electricity markets because of the very specific features present in electricity spot prices. These characteristics are strong seasonality, mean reversion, jumps, stochastic volatility and regime switching (see Escribano, Peña and Villaplana (2011) among others) and are caused by the difficulties of storing electricity efficiently; see also Lucia and Schwartz (2002) and Cartea and Figueroa (2005). Besides that, this approach has some disadvantages such as its inability to incorporate information about the future (e.g. addition of new generation facilities) and because endogenously generated forward prices are not necessarily consistent with observable forward prices. Quinn, Reitzes and Scumacher (2005) argue that electricity forward prices are a function of market expectations of demand and cost conditions during the actual delivery period, and these expectations are not necessarily influenced by current market behaviour (i.e. spot prices), and they present evidence supporting their claim in the PJM market. Furthermore, the historical correlation between electricity spot prices and the nearby futures prices is not particularly stable, which suggests that the spot price is not a good proxy for the futures prices. Benth, Šaltyte-Benth and Koekebakker (2008) find that contracts located in the very short end of the forward curve are the only ones with sizeable correlations with the spot price at Nord Pool. Borovkova and Geman (2006b) document that, for the Nord Pool data, the historical correlation (computed by means of a moving window of the past 60 days) between spot prices and the nearby monthly futures contract ranges from 0.65 to -0.15. In our EEX data sample, and using the same procedure, the average value of this correlation is 0.34, but it ranges from 0.87 to -0.54[3]. To put these figures into perspective, Alexander (1999) reports that the average correlation between WTI crude oil spot and NYMEX near monthly futures prices is 0.83 and it ranges from 0.65 to 0.95.

The third category is based on the direct modelling of the term structure of the electricity forward prices. Within this category, a first line of research is based on the spirit of Heath, Jarrow and Morton (1992) (HJM) which focus on the dynamics of the forward curve as a whole. Examples of this approach are Cortazar and Schwartz (1994), Amin, Ng, and Pirrong (1995), Miltersen and Schwartz (1998), Clewlow and Strickland

[3] In the case of log returns, the average value of the correlation is -0.005, and the correlation ranges from 0.45 to -0.35.

(1999), Koekebakker and Ollmar (2005), Miltersen (2003), Keppo, Audet, Heiskanen, and Vehviläinen (2004) and Trolle and Schwartz (2009), among others. In all these cases the market forward price curve is an input into the derivative pricing model and therefore derivatives prices thus generated should be consistent with observable forward market prices. A second line of research is based on modelling a given function of observed forward prices and then analysing stochastic deviations from this function by means of additional state variables. An example of this approach is Borovkova and Geman (2006a), who propose to use a parsimonious two-factor model, in which the first factor is the average forward price and the second factor is analogous to the stochastic convenience yield. However, a potential problem of all the previous models within this category is their assumption of Gaussian distributions for the innovations of the stochastic processes[4]. As suggested by Frestad, Benth and Koekebakker (2010) in the case of electricity futures, this assumption is unlikely to be appropriate, because the innovations of the electricity forward prices are strongly non-normal. Besides that, the factor structure of the forward curve in the electricity markets is probably much more complex than in another energy markets. For instance, Koekebakker and Ollmar (2005) report that, in order to explain more than 98% of the variation in the sample covariance matrix, more than ten factors were needed. Interestingly, factors explaining a large proportion on the variation in the long end of the curve, seem to have very low explanatory power in the short end of the curve, which suggests that, besides a few general factors affecting the whole curve, some parts of the curve are exposed to unique risk factors, that other parts of the curve are not exposed to. This idea of common and unique factors is explored in Frestad (2008) who find strong support for this hypothesis in the Nordic electricity market.

For these reasons, we propose a model for the term structure of the electricity swap prices that follows the parsimonious second line of research outlined above, but allowing for a very general distribution for the sources of uncertainty (innovations) impacting the state variables, in the spirit of the common and unique factors described above. In a nutshell, our model features factors accounting for (a) the average forward price within each market segment, (b) the deterministic seasonal factor and, (c) the stochastic changes in the forward curve shape, and is based on a particular case of the

[4] An exception is Andresen, Koekebakker and Westgaard (2010) who present a discrete random-field model based on the multivariate NIG distribution

Multivariate Generalized Hyperbolic (MGH) distributions, namely the Multivariate Normal Inverse Gaussian (MNIG) distributions, which allow stochastic dynamics in terms of correlated NIG Lévy processes. There is growing evidence suggesting that the normal inverse Gaussian (NIG) distribution, which was first used for modelling speculative returns in Barndorff-Nielsen (1997), fits heavy-tailed and skewed financial data well and is, at the same time, analytically tractable, see, for instance, Rydberg (1999), Barndorff-Nielsen and Prause (2001), Forsberg and Bollerslev (2002), and Karlis (2002) among others. Therefore the NIG distribution is especially suitable for the modelling of financial prices, and in particular for the term structure individual contract dynamics (Benth, et al, 2008) as well as their joint evolution (Andresen, Koekebakker and Westgaard (2010)).

The contribution of this article is threefold. First, our model captures the basic stylized facts of swap curves in electricity markets, and in particular their volatilities, correlations, asymmetries and kurtosis, by means of an analytically tractable model specification. Second, we present evidence supporting the notion that our model offers better empirical fit than spot price-based or traditional HJM-based models. Third, we show that the specific component in each swap contract represents a non-negligible source of risk. This risk is specific of each contract and cannot be hedged by using other swap contracts. The consequence of this is that a trader wishing to hedge long-term swap contracts (e.g. yearly) using shorter-term contracts (e.g. quarterly) in the EEX market is likely to be exposed to basis risk of significant importance. Fourth, we show that, models who do not account for the impact of non-normality are not able to replicate market prices, and, in particular, they are unable of taking account of tail risk. This result is important because Gonzalez-Pedraz, Moreno and Peña (2014) present evidence suggesting that tail risk measures for energy portfolios based on standard methods (e.g. normality) and on models with exponential tail decay underestimate actual tail risk, especially for short positions and short time horizons. Overall, the results suggest that our model provide extra explanatory power in comparison with these alternatives.

This study extends current literature in several ways. First, Kiesel, Schindlmayr and Börger (2008) suggest a two-factor model for electricity futures calibrated to at-the-money options on electricity futures traded in the EEX market. However these options are not particularly liquid. Thus, we choose instead to work directly with the swap prices, because, at least some of them, are usually highly liquid. Second, in contrast with Benth and Koekebakker (2008) who allow for just one Brownian motion as the driver of the dynamics of the forward, we argue that correlated Lévy process for each market segment should be introduced to explain the dynamic behaviour of swap prices, allowing for a more realistic representation. Third, Fleten and Lemming (2003), Keppo, Audet, Heiskanen and Vehviläinen (2004), Koekebakker and Ollmar (2005) and Bjerksund, Rasmussen, and Stensland (2010) build a continuum of instantaneous-delivery forward contract by smoothing market prices, or by combining market prices with forecasts generated by bottom-up models. However, we argue that working directly with the most liquid swap prices is more transparent, instead of using ad hoc numerical procedures to extract smooth curves from quoted prices[5]. In our case the most liquid contracts are the six contracts closer to maturity for the monthly, quarterly and yearly delivery periods.

Summing up, our contributions are as follows. First, we propose a new stochastic forward premium model that includes exposures to average forward prices within each market segment, deterministic seasonal factors and mean-reverting stochastic deviations from the average price following MNIG processes. This model effectively captures realistic, time-varying characteristics in forward prices, overcoming the limitations of standard models of forward curves that cannot account for asymmetries and fat tails. Second, the results suggest that the specific component in each swap contract represents a non-negligible source of risk, which is specific of each contract and cannot be hedged by using other swap contracts. The consequence of this is that traders in the EEX market are likely to be exposed to basis risk of significant importance. Third, our empirical results based on the EEX market during the period from 2004 to 2013 provide strong evidence supporting our model in comparison with other alternatives. A particularly important practical implication of our VaR analysis is that that the capital charges to traders in the EEX (based on risk adjusted capital under the normality assumption) should be adjusted (most likely upwards) and that the evaluators of the performance of

[5] Advocates of the smoothing algorithms posit that futures prices with fixed time to maturity can be extracted each day from the smoothed curve. The main criticism is however that the prices are not "true" market prices but interpolations and these smoothed prices may distort the empirical analysis. We prefer to concentrate on actual market prices and include the effect of the changing time to maturity in the SFP component

the traders should adjust their recommendations accordingly.

The rest of paper is organized as follows. Section 2, we derive our model. After we describe the data in Section 3, we report the results of the empirical analysis in Section 4. Section 5 compares the model against alternative approaches. Section 6 presents some Value-at-Risk results. Section 7 concludes.

2. The Model

In this section we outline the basic characteristics of the theoretical model and present some guidelines for its practical implementation. We will also make use of the MNIG distribution and we refer to readers to McNeil, Frey, and Embrechts (2005) for definition and properties of this distribution.

2.1 The general model

Assume T<∞ and let (Ω,F,Q) be a complete filtered probability space, with an increasing and right-continuous filtration $\{F_t\}_{t\in[0,T]}$ where, as usually, F_o contains all sets of probability zero in F. We assume that the market trades forward contracts with different delivery periods and a bond that yields a constant risk free rate r >0, so futures and forward prices are equal. Consider the price $F_i(t,T)$ of a forward contract with expiry date $T = 1, \dots, N$, which is also the start of the delivery period, time to maturity $\tau = (T - t)$,and delivery length period given by the subscript $i = 1, \dots, I$ (e.g. i=1 (M) for monthly contracts, i=2 (Q) for quarterly contracts and i=3(Y) for yearly contracts)[6]. These contracts are settled against the daily average spot price during the delivery period and, in agreement with market practice, we call them electricity swaps. Hence, $F_{\boldsymbol{i}}(t,\boldsymbol{T})$denotes the price vector of a completely observable swap curve at time t, where vector $\boldsymbol{T}$ indicates the different swap contracts starting delivery dates which are available at trading date t, and the vector subscript $\boldsymbol{i}$ denotes the underlying asset (delivery period) over which is defined each curve's swap contract. Notice that in a general term structure model, it is considered the evolution of the continuum of swap

[6] For example, the swap price $F_M(t,1)$ is the price, at time t, of the contract that matures at the end of the current month (e.g. January) and provides delivery of electricity at a fixed price during the next month (e.g. February). This contract is the M+1 or M1 monthly contract in market parlance. Similarly, $F_Q(t,2)$ is the price of the Q2 quarterly contract, and $F_Y(t,3)$ is the price of the Y3 yearly contract.

contracts for all the possible expirations. Since only subsets of them are actually traded in the market, we develop a version of the model that specifies the dynamics only for tradable contracts which are liquid enough. Therefore our model follows the spirit of the market model, see Brace, Gatarek, and Musiela (1997), which was originally developed for interest rates. A similar model to ours is proposed in Borovkova and Geman (2006b), but they consider only two sources of uncertainty, driven by uncorrelated Gaussian innovations. We instead allow for multiple sources of uncertainty, and these sources of uncertainty follow correlated MNIG distributions. In doing, so our model captures the correlation among the sources of uncertainty, as well as some salient stylized facts in the swap electricity market, such as extreme kurtosis. The starting point of our analysis is the following stochastic forward premium model (SFP henceforth), which relates swap prices for any maturity T and delivery length period i as follows,

$$F_i(t,T) = \bar{F}_i(t)e^{(s_i(K)+\gamma_i(t,T))} \tag{1}$$

where the first component in the right-hand side is the average level of the swap price within each market segment i defined as the geometric average of the current swap prices as follows

$$\bar{F}_i(t) = \sqrt[N]{\prod_{T=1}^{N} F_i(t,T)} \tag{2}$$

and N is the maximum liquid maturity[7]. The average swap price does not contain seasonal factors, which are included in the deterministic seasonal premia factor $s_i(K)$. This premia is defined as the collection of long-term average premia on swaps expiring in the calendar period (week, month, quarter) with respect to the average swap price. Depending on the value of i, K can take 52 (weekly), 12 (monthly) or 4 (quarterly) different values. By construction the average of the seasonal factors for each i must be zero. For instance in the case of which i=M, that is a monthly (M) delivery period, there are twelve seasonal factors $s_M(K)$, K=1,…,12 that is, a deterministic collection of 12 parameters.

[7] For instance if there are three delivery periods, the first with a length of one month, the second with a length of three months (one quarter) and the third with a length of twelve months (one year), thus i= 1(M),2(Q),3(Y) and I=3. If there are six liquid maturities 1 to 6, then T=1,2,…,6 and N=3. In summary, the liquid contracts are M1,M2,M3,M4,M5,M6, Q1,Q2,Q3,Q4,Q5,Q6 and Y1,Y2, Y3, Y4,Y5 and Y6.

The quantity $\gamma_i(t,T)$ is the stochastic forward premium[8] (SFP) for the delivery period *i* and expiry date *T*. By construction this SPF is zero on average, and it is defined as

$$\gamma_i(t,T) = lnF_i(t,T) - ln\overline{F_i}(t) - s_i(K) \quad (3)$$

Next we specify the stochastic dynamics of the state variables in terms of Multivariate Normal Inverse Gaussian (MNIG) Lévy processes. Notice that the dimension of the system is d = I+ I×N. Let $\boldsymbol{L}(t)$ be a d-dimensional vector of MNIG Lévy processes. This means that it has stationary and independent increments in the sense that the distribution of $\boldsymbol{L}(t) - \boldsymbol{L}(s)$, $t > s \geq 0$, is only dependent on *t-s* and not on *t* and *s* separately, such that its increments $d\boldsymbol{L}(t)=\boldsymbol{L}(t+dt) - \boldsymbol{L}(t) = \boldsymbol{X}$ are[9], standardized (i.e. zero-mean, unit variance) and MNIG distributed with probability density function $MNIG_d(\boldsymbol{X};\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$

$$f(\boldsymbol{X}) = \frac{\delta}{2^{(d-1)/2}} \left[\frac{\alpha}{\pi q(\boldsymbol{x})}\right]^{(d+1)/2} K_{\frac{d+1}{2}}[\alpha q(\boldsymbol{x})] \times e^{p(\boldsymbol{x})} \quad (4)$$

where

$$q(\boldsymbol{x}) = \sqrt{\delta^2 + (\boldsymbol{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})} \quad , \quad p(\boldsymbol{x}) = \delta\sqrt{\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}} + \boldsymbol{\beta}'(\boldsymbol{x}-\boldsymbol{\mu})$$

and $K_{\frac{d+1}{2}}[x]$ is the modified Bessel function of the second kind with index *(d+1)/2*, and the parameters have the following characteristics, $\delta > 0$, $\alpha^2 > \boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}, \boldsymbol{\beta} \in \mathbb{R}^{\boldsymbol{d}}, \boldsymbol{\mu} \in \mathbb{R}^{\boldsymbol{d}}, \boldsymbol{\Sigma} = \{\sigma_{ij}\} \in \mathbb{R}^{\boldsymbol{d}\times\boldsymbol{d}}$ and we require[10] $\boldsymbol{\Sigma}$ to be positive definite and $|\boldsymbol{\Sigma}| = 1$. The mean vector of $\boldsymbol{X}$ is

[8] This SFP can also be written as $\gamma_i(t,T-t)(T-t)$ as in Borovka and Geman (2006b) to suggest that the effect of the time to maturity *(T-t)* enters the futures price via the SPF. However, for the sake of clarity, we choose the more compact notation but stress the fact that the time-to-maturity effect is included in the SFP component.

[9] In the special case where the increments $\boldsymbol{L}(t)-\boldsymbol{L}(s)$ are normally distributed with zero mean and covariance matrix Π, we have a standard multivariate Brownian motion.

[10] The distributions $MNIG_d(\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $MNIG_d(\alpha/\kappa, \kappa\boldsymbol{\beta}, \kappa\delta, \kappa\boldsymbol{\mu}, \kappa\boldsymbol{\Sigma})$ are identical for any $\kappa > 0$ Therefore, an identifying problem occurs when we start to fit the parameters of a MNIG distribution to data. This problem is solved by introducing a suitable constraint, for instance requiring that the determinant of the dispersion matrix $\boldsymbol{\Sigma}$ is equal to one.

$$E[\boldsymbol{X}] = \boldsymbol{\mu} + \delta\boldsymbol{\Sigma\beta}\sqrt{\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta}}$$

and the covariance matrix $V[\boldsymbol{X}] = \{v_{i,j}\}\ i = 1, \dots, d\ ; j = 1, \dots, d$ is defined as

$$V[\boldsymbol{X}] = \delta\sqrt{\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta}}[\boldsymbol{\Sigma} + (\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta})^{-\mathbf{1}}\boldsymbol{\Sigma\beta\beta}'\boldsymbol{\Sigma}] \tag{5}$$

Note that if the skewness parameter is zero ($\boldsymbol{\beta} = 0$) the mean vector coincides with $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ solely determines the correlation structure. For asymmetric MNIG distributions, the correlation structure depends on all parameters, excepting $\boldsymbol{\mu}$.

The marginal distributions of the MNIG distribution are univariate NIG distributions (Lillestøl, 2000). Denoting the parameters of the marginal distributions of the i th component of $\boldsymbol{X}$ as X_i and using the obvious notation, we have $\mu_i = \boldsymbol{\mu}_i$, $\delta_i = \delta(\sigma_{ii})^{1/2}$, $\beta_i = \sigma_{ii}^{-1}\sum_{k=1}^{d}\sigma_{ik}\boldsymbol{\beta}_k$ and $\alpha_i = \sqrt{\sigma_{ii}^{-1}(\alpha^2 - \boldsymbol{\beta}'\boldsymbol{\Sigma\beta}) + \beta_i^2}$. The scale-free indicators of asymmetry χ_i and kurtosis ξ_i , {(χ_i , ξ_i) $\in R^2; |\chi_i| < \xi_i < 1$) are respectively defined as (Rydberg, 1997)

$$\chi_i = \frac{\xi_i\,\beta_i}{\alpha_i}\ ;\ \xi_i = \frac{1}{\sqrt{(1+\delta_i\gamma_i^2\,)}}$$

where $\gamma_i = \sqrt{\alpha_i^2 \quad - \beta_i^2}$. If (χ_i , ξ_i) ≈ (0,0) the marginal NIG distribution is close to being normal. On the other hand, the limit (ξ_i) ≈ (1) gives the heavy-tailed Cauchy distribution.

We define de dynamics of $ln\overline{F_i}(t)$ and $\gamma_i(t,T)$ under the market probability measure by the stochastic differential equations

$$dln\overline{F_i}(t) = \kappa_i\big(\zeta_i - ln\overline{F_i}(t)\big)dt + \theta_{\overline{F,i}}dL_{\overline{F_i}}(t)\ ;\ i = 1, \dots, I \tag{6}$$

$$d\gamma_i(t,T) = \varpi_{i,T}\gamma_i(t,T)\,dt + \theta_{\gamma_i(T)}dL_{\gamma_i(T)}(t)\ ;\ i = 1, \dots, I\ \ T = 1, \dots., N \tag{7}$$

The SFPs are subject to their own sources of uncertainty, given by the standardized MNIG Lévy processes, $dL_{\overline{F_i}}(t)$, and $dL_{\gamma_i(T)}$ which are assumed to be correlated. We can substitute (6) and (7) into (1) and derive the dynamics of the futures log-prices under the market probability measure as follows

$$dln\big(F_i(t,T)\big) = \big[\kappa_i\big(\zeta_i - ln\overline{F_i}(t)\big) + \varpi_{i,T}\gamma_i(t,T) + s_i(K)\,\big]dt + \theta_{\overline{F,i}}dL_{\overline{F_i}}(t)$$
$$+\ \theta_{\gamma_i(T)}dL_{\gamma_i(T)}(t)$$
$$i = 1,\dots,I\ \ T = 1,\dots.,N$$

And therefore $lnF_i(t,T)$ is obtained by integrating the above differential equation with the initial condition[11]

$$lnF_i(0,T) = ln\overline{F_i}(0) + s_i(K) + \gamma_i(0,T)\quad i = 1,\dots,I\ \ T = 1,\dots.,N$$

The term structure of swap prices variances is given by

$$\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,i}}\theta_{\gamma_i(T)}v_{\overline{F_i},\gamma_i(T)}\ ;\ \ i = 1,\dots,I\ ;T = 1,\dots,N \qquad (8)$$

where $\eta_i^2 = {\theta_{\overline{F,i}}}^2 \times v_{\overline{F_i,F_i}}$ and $\tau_{i,T}^2 = {\theta_{\gamma_i(T)}}^2 \times v_{\gamma_i(T),\gamma_i(T)}$ are the variances of the corresponding average factor and of the stochastic discount factor respectively and $v_{\overline{F_i},\gamma_i(T)}$ is the covariance between the average factor and the stochastic discount factor, all of them elements of (5).

If we are interested in pricing other derivatives, we now consider how to price these derivatives in the risk-neutral world. First, notice that the dynamics (1), (6) and (7) are specified under the market (real-world) probability measure *P*; therefore we must select a risk-neutral probability measure *Q*. A common choice (see Benth, Šaltyte-Benth and. Koekebakker , 2008) is the Esscher transform which generalizes the Girsanov transform to Lévy processes and guarantees that the ***L(t)*** process is still a NIG Lévy

[11] If X_1 and X_2 are independent NIG random variables with common parameters α,β but having different scale and location parameters δ_1,μ_1, and δ_2, μ_2, then $X_1+X_2 = X$ is NIG with parameters $(x;\alpha,\beta\ \delta_1+\delta_2, \mu_1+\mu_2)$.

process under Q. This Esscher transform implies that, under the risk-neutral measure Q, the transformed $\boldsymbol{L}^{Q}(t)$ vector is MNIG-distributed with density function $MNIG_d(\boldsymbol{X}; \alpha, \boldsymbol{\beta}+\boldsymbol{\eta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$, or in other words, the Esscher transform only changes the asymmetry of the process. The vector $\boldsymbol{\eta}$ measures the price of jump risk, that is, the price that market players charge for assuming the risk of not being able to hedge. A positive price leads to a more right-skewed distribution. Given an estimate[12] of the vector $\boldsymbol{\eta}$, and the transformed process under Q, standard techniques can be applied to price other derivatives such as options.

2.2. Implementation

Assume we have an historical dataset of n daily swap curves $F_{\boldsymbol{i}}(t, \boldsymbol{T})\ \ t = 1, \ldots n$, where vector $\boldsymbol{T}$ indicates the different swap contracts starting delivery dates which are available at trading date t, and the vector subscript $\boldsymbol{i}$ denotes the underlying asset (delivery period) over which is defined each curve's swap contract. Writing (1) in logarithm form

$$lnF_i(t, T) = ln\overline{F_i}(t) + s_i(K) + \gamma_i(t, T) \tag{9}$$

The least squares optimal estimator for $ln\overline{F_i}(t)$ is simply the arithmetic average of log-swap prices within each market segment i. We estimate factor $s_i(K)$ by

$$\widehat{s_i}(K) = \frac{1}{n} \sum_{T_{k \in A_{(i)}}} \sum_{t=1}^{n} (\ln F_i(t, T_k) - ln\overline{F_i}(t)\ ) \tag{10}$$

where $A_{(i)}$ are the sets of available maturities at time t for contracts with delivery period i=1,...,I, and the index K takes different values k.[13] The seasonal factors must be zero on average for each seasonal period (e.g. monthly) , and then $\sum_{k=1}^{K} s_i(k) = 0$. We estimate the SFP factor by means of the equation

$$\widehat{\gamma_i}(t, T) = \ lnF_i(t, T) - ln\overline{F_i}(t)\ -\ \widehat{s_i}(K) \tag{11}$$

For the sake of clarity, we use the notation i=*1(M),2(Q),3(Y)* and T=*1,…,6* in

[12] Estimation methods in the case d=1 are proposed in Benth et al (2008) and in Frestad, Benth and Koekebakker (2010)..

[13] This index refers to each month (k=1,…,12, K=12), or quarter(k=1,…,4, K=4)

what follows. Given the complexity of the estimation we apply a two-step procedure. In the first step we estimate simultaneously all the parameters of mean reversion and volatility in discrete-time versions of equations (6) and (7) by means of a system of seemingly unrelated regression equations (SURE). In doing so, we assume that error terms may have cross-equation contemporaneous covariance[14]. The system takes the form[15]

$$\begin{pmatrix} \nabla ln\overline{F_i}(t) \\ \nabla \hat{\gamma_i}(t,T) \end{pmatrix} = \begin{pmatrix} \kappa_i(\zeta_i - ln\overline{F_i}(t-1)\ ) \\ \varpi_{i,T}\hat{\gamma}_i(t-1,T) \end{pmatrix} + \begin{pmatrix} \theta_{\overline{F_i}}\varepsilon_{\overline{F_i}}(t) \\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t) \end{pmatrix} \tag{12}$$

i=1(M),2(Q),3(Y)   T=1,…,6   t=1,…,n

The system contains 21 equations (3+6×3) and this SURE model is estimated using the feasible generalized least squares (FGLS) method. Given the dimensions of the problem and that in the second step we require the covariance matrix of the MNIG process $\mathbf{\Sigma}$ to be positive definite and $|\mathbf{\Sigma}| = 1$ a convenient normalization suggested in Urzua (1997) is as follows. Let the residuals form equation (12) be defined as

$$\boldsymbol{Y} = \begin{pmatrix} \varepsilon_{\overline{F_i}}(t) \\ \varepsilon_{\gamma_i(T)}(t) \end{pmatrix}\ t = 1, \ldots, n \tag{13}$$

where $\boldsymbol{Y}$ (dimensions d×n) has a mean vector of zero and covariance matrix $\Omega = \{\omega_{i,j}\}$ Let Γ denote the orthogonal matrix whose columns are the standardized eigenvectors of Ω, and Λ denote the diagonal matrix of the eigenvalues of Ω . Define $\Omega^{-1/2}$ as the inverse of the square root decomposition of Ω ; or, in other words, that

$$\Omega^{-1/2} = \Gamma\Lambda^{-1/2}\Gamma' \tag{14}$$

Then the random variable

$$\boldsymbol{X} = \Omega^{-1/2}\boldsymbol{Y} \tag{15}$$

has a zero mean vector, and an identity matrix as its covariance matrix. This variable $\boldsymbol{X}$, which contains the standardized and orthogonal residuals, is then used in the second step.

[14] We include in the estimation appropriate AR terms to take into account possible residual autocorrelation.
[15] $\nabla x(t) = x(t) - x(t-1)$

In the second step we use $\boldsymbol{X}$ as the estimation of the vector $d\boldsymbol{L}(t)$, which is assumed to be MNIG distributed with probability density function $\text{MNIG}_d(\alpha, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ and we estimate the corresponding parameters of the MNIG distributions by using the is the EM algorithm developed by Øigård, Hanssen, Hansen, and Godtliebsen (2005) which is an extension of Karlis (2002)[16] . The bootstrapped confidence intervals are based on 500 replications. In order to compute the term structure of swap prices given by (8) we set $\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,\iota}}\theta_{\gamma_i(T)}\omega_{\overline{F_\iota},\gamma_i(T)}$ where $\eta_i^2 = \theta_{\overline{F,\iota}}{}^2 \times \omega_{\overline{F_\iota,F_\iota}}$ and $\tau_{i,T}^2 = \theta_{\gamma_i(T)}{}^2 \times \omega_{\gamma_i(T),\gamma_i(T)}$ are the variances of the corresponding average factor and of the stochastic discount factor respectively and $\omega_{\overline{F_\iota},\gamma_i(T)}$ is the correlation between the two factors, all of them obtained from (12).

3. **Data**

Our data set consists of daily data from June 1, 2004 until December 31, 2012, on settlement prices for the following available baseload[17] swap contracts traded in EEX: Yearly baseload, Quarterly baseload and Monthly baseload. The company operating EEX market (EEX AG) has provided the data.[18] We choose the six most liquid contracts within each market segment, that usually are the closest to maturity ones. Within each market segment, these six contracts represent the 99% (100%), 97% (99%) and 100% (100%) of the total trading volume (open interest) in the case of monthly, quarterly and yearly contracts, respectively. The continuous series are defined as a perpetually linked series of swap settlement prices. For example, M1 starts at the nearest contract month, which forms the first values for the continuous series until either the contract reaches its expiry date, or until the first business day of the actual contract month. At this point the next trading contract month is taken. [19] For all series we compute the returns as the first

[16] An alternative method is the Multi-cycle Expectation Conditional Maximization (MCECM) algorithm developed in McNeil, Frey, and Embrechts (2005).

[17] As an illustration of this kind of contracts, the 1MW baseload Jan13 contract is a monthly swap contract that gives the holder the obligation to buy 1MWh of energy for each hour of January 2013, paying the futures price in Euros/MWh. The seller provides the buyer the amount of energy of 1MW × 24h × 31. The settlement is financial.

[18] The futures market at the EEX started trading financial futures on base and peak block contracts in the spring of 2001. In 2004 option trading on these contracts was introduced, and since 2005 futures with physical settlement were introduced. Other basic facts on the EEX futures market can be found in for instance in European Energy Exchange (2005), see also Viehmann (2011).

[19] The continuous series M1, Q1 and Y1 match the series from Datastream: EBMCS00, EBQCS00 and

difference of log prices.[20]

3.1 Summary Statistics

The graphs for all swap price series are shown in Figure 1 by market segments. The monthly series seem to be the more volatile followed by the quarterly series, being the yearly series the most stable one.

[INSERT FIGURE 1 HERE]

Table 1 provides information on the basic statistics for all series in levels. The individual series are shown in Panel A and the averages of each market segment are shown in in Panel B. Looking at the averages of each market segment (monthly, quarterly and yearly), the average price tends to increase with the maturity of the contract and the volatility tends to decrease with the maturity of the contract, as expected. Volatility is usually higher for the closest-to-maturity contract (Samuelson effect), confirming the well-known fact that short-dated swaps tend to be more volatile than long-dated swaps. Within each class, the average price follows the same pattern, but the volatility has more complex behaviour. In the case of monthly contracts, the volatility has an inverted u-shape, in the case of quarterly contracts, it has a u-shape and in the case of yearly contracts it increases with time to maturity. In the returns series (not shown) the volatility follows the expected pattern, decreasing with time to maturity and ranging (in annualized terms) from 33% in the case of M1 to 13% in the case of Y6. All series present positive asymmetry and significant kurtosis, suggesting that the normality assumption is unlikely to be appropriated for these series.

[INSERT TABLE 1 HERE]

EBYCS00.

[20] To build the continuous series, a level change appears on the day when one contract expires and a new one is included. This is the so called "rolling" effect which sometimes generates jumps in the returns series. So, to avoid this artificial effect in the returns series, we apply intervention analysis (see Box and Tiao, 1975) in each day when there is a “rolling” effect. Notice that these jumps are not caused by market behavior, but are simply a technical problem caused by the definition of the continuous series.

4. Empirical Results

In this section we present the estimation of the components of the swap price as defined in equation (1), that is, the average forward price, the seasonal components and the stochastic forward premium.

4.1 Average forward prices

We compute average forward prices as the simple average of swap log prices within each market segment. Figure 2 contains the graphs and the basic statistics are in Table 2.

[INSERT FIGURE 2 HERE]

[INSERT TABLE 2 HERE]

The basic statistics of the average forward price are consistent with the ones in Table 1 in the sense that average price increases with maturity and volatility decreases with maturity. The average return is close to zero, the volatility is higher in the monthly segment and it is lower in the yearly segment, and all series have positive asymmetry and high kurtosis. It is interesting to note that the correlation (both in levels and in first differences) is far from one in the case of the averages in the monthly and yearly segments. This fact stresses the convenience of working with a model based on a specific average component for each market segment.

4.2 Seasonal Components

The seasonal component computed using the method described in equation (10) is shown on Figure 3.

[INSERT FIGURE 3]

As expected, swaps expiring in fall and winter are at a premium with respect to the average price level, and swaps expiring in spring and summer are at a discount. The January and February premium is the highest, at 9%. On the other hand, May has the highest discount, at 11%. A statistical significance test (not shown) reveals that most seasonal components are significantly different from zero at the usual levels.

4.3 Stochastic forward premium factors

Using equation[21] (11) we compute the estimated SFPs. Their basic statistics are in Table 3, Panel A. All $\widehat{\gamma}_i(t,T)$ series have means which are close to zero, as expected, and volatilities generally fall as more distant market segments are considered, furthermore, all series present some asymmetry and kurtosis, as well as first order autocorrelation coefficients close to one (not shown). All series $\widehat{\nabla\gamma_i}(t,T)$ present means which are essentially zero, and volatilities which usually decrease with the time to maturity. Some present positive skewness but others present negative skewness, and the first-order autocorrelation coefficient (not shown) is usually below 0.1, suggesting a slow mean-reverting behaviour. High kurtosis seems to be a salient feature in all cases. Therefore the evidence suggests that all these series are highly non-normal.

[INSERT TABLE 3 HERE]

The correlations between the returns of the average series and the returns of the SFPs series are shown in Table 4. The monthly contracts tend to be correlated with their average factor and with nearby monthly contracts as well as with some quarterly contracts. However, their correlation with the yearly contracts is usually not high. The yearly contracts tend to be correlated among them and also are correlated with its average factor but their correlation with the monthly and quarterly contracts is usually low. Overall, the evidence is consistent with the assumptions in our theoretical model, because it includes correlations both within and across market sectors, and thus it permits a more realistic representation of market prices.

[INSERT TABLE 4 HERE]

4.4 The relative weight of the components

An important practical question is the proportion of the total variation explained by each component for the different market segments. To study this issue we run a regression,

[21] All SPF series are adjusted to discard the rolling effect. See footnote 14.

where the explanatory variables are the components which are included sequentially. In the monthly segment, on average, the average forward price explains around 75% of the total variation, the SFP component explains an additional 15% and the seasonal component explains the remaining 10% of the total variation, and the differences across maturities are not very marked. The same situation appears in the quarterly segment, where the average forward price explains around 80%, the SFP component explains an additional 5% and the seasonal component explains the remaining 15% of the total variation. In the case of the yearly segment, and on average, the average forward price explains around 85%, and the SFP component explains the remaining 15% of the total variation[22].

One implication of these results is that the specific component in each swap contract represents a non-negligible source of risk. This risk is specific of each contract and cannot be hedged by using other swap contracts. For instance if one trader wants to hedge a position in a yearly contract using monthly contracts, there is an specific risk that can be as high as more than 20% of total risk, and this idiosyncratic risk cannot be avoided by using monthly contracts. Additionally, by using these monthly contracts, the trader is assuming additional idiosyncratic risks in each monthly contract. The consequence of this is that a trader wishing to hedge swap contracts in the EEX market, is likely to be exposed to a significant basis risk.

4.5 Model Estimation

In Table 5 we present the results of the results of the estimation of model (12) for the average swap prices and the stochastic discount factors. Panel A contains the results for the average series factor and the estimation of the parameters of their individual NIG distributions. Panels B,C and D contain the same information for the SFP of each contract within each market segment (monthly, quarterly and yearly). Panel E contains the estimation of the parameters of the MNIG distribution of the standardized and orthogonalized residuals obtained from equation (12). In Panels A, B, C, and D the

[22] In the yearly segment some more noticeable differences across maturities appear, because the average forward price explains around 93%, 85% and 78% for the one year, two year and three year maturities respectively, and the SFP component explains the additional 7%, 15% and 22%.

mean reversion parameter is significant in all cases, suggesting a mean reverting process, albeit with different speeds. The process is faster in the case of the SPF of the monthly contracts, followed by quarterly and then yearly contracts, being the slowest in the case of the processes for the average prices. The residual volatility varies considerably, being higher in the case of the residuals of the processes for the average prices and decreasing with the time to maturity in the case of the SPF across market segments, as expected.

A test of multivariate Normality (Urzua,1997) of the residuals (not show) clearly reject the null of normality. Regarding the NIG parameters of the individual distributions, estimated α's are always significant and vary from 0.44 (Y6) to 1.04 (Y2) , the β's are usually positive but non-significant, the δ's are positive and significant, ranging from 0.43 to 1.03 (same contracts as before) , and the μ's are very close to zero, as expected. The scale-free indicator of asymmetry, χ varies around 0.03 but usually it is not significant and the kurtosis parameter ξ is highly significant and varies from 0.68(Y2) to 0.95 (Y6). The likelihood ratio test[23] strongly supports the NIG distribution as a better alternative than the standard normal distribution for all individual distributions. It is also interesting to note that the SFP component corresponding to each swap contract, shares some common characteristics with the other SFPs within each market segment (symmetry, the degree of mean reversion) but it also has some specific features (different volatility levels).

The parameters of the MNIG distribution shown in Panel B give a similar message, with non-significant mean and asymmetry parameters and significant values in the cases of the parameters measuring tail heaviness and scale. The overall impression is that the MNIG distribution can be characterized as being essentially symmetric and having strong tail heaviness

Table 6 contains the correlations among the ordinary residuals from equation (12) and in boldface we highlight the correlations used in the computations of the volatility term structure.

[INSERT TABLE 5 HERE]

[23] *LR = -2*LOGLK(N(0,1)) + 2*LOGLN(NIG(α,β,δ,μ))*

[INSERT TABLE 6 HERE]

4.6 In-sample Goodness-of-Fit

As a formal test of the extent to which the NIG distribution is successful in representing the (one-day) innovations in the marginal distributions of the average forward price and the SFPs, we implement tests of fit based on the empirical distribution function (EDF). These statistics measure the discrepancy between the EDF and a given theoretical distribution (e.g Normal or NIG). We calculate two statistics: the Cramer-von Mises and the Kolmogorov-Smirnov. [24] We calculate the parameters of the NIG distribution by maximum likelihood. Next, we face the problem of how to evaluate these test statistics, given that the true parameter values of the (NIG) distribution are unknown. To solve this problem, and following Capasso, Alessi, Barigozzi and Fagiolo (2009), we generate 5000 Monte Carlo simulations of i.i.d. NIG random numbers for each market segment. In doing so, we obtain approximate distributions of the EDF test statistics. We briefly summarize the results. The null hypothesis is that the stochastic elements in the all marginal distributions can be represented by NIG distributions. The number of violations of the null hypothesis at the 5% significance level is always lower than the critical value suggesting that, in all cases, one-day returns are well represented by the NIG distribution.

4.7 Volatility term structure

In Table 7 we present the volatility term structure that has been computed by using Equation (8) and (12) with the calibrated parameters obtained in Table 5 and Table 6.

[INSERT TABLE 7]

As may be seen in Table 7 the model is able to reproduce the overall volatility

[24] For details on EDF statistics see D’Agostino and Stephens (1986).

structure with high degree of precision irrespective of the market segment. Average absolute and relative errors are lower than 0.5%. The average contributions of each of the elements in (8) to the total variance explained by the model are as follows: 87.5% corresponds to the variance of the corresponding average factor, 18.5% corresponds to the variance of the stochastic discount factor, and -6.1% corresponds to the covariance term between the average factor and the stochastic discount factor. However the range of variation is substantial, being in the first case from 41% to 111%, in the second case from 9% to 37% and in the third case from -49% to 31%. The implication of these results is that the order of magnitude of determinants of the variance for each contract should be analysed carefully because, although in some cases the impact of the average factor dominates, in other cases the impact of the covariance component can be determinant.

4.8 Hedging effectiveness

One key implication of the results of our model when applied to EEX data is that the specific component in each swap contract represents a non-negligible source of risk. To analyze the extent to which this risk is specific of each contract and cannot be hedged by using other swap contracts, we briefly discuss the results obtained from a standard measure of hedging effectiveness, namely the one which is based on the adjusted R^2 produced by a regression in which the change in the value of the hedged item (e.g. a long-term swap contract) is the dependent variable and the change in the value of the hedging variable (e.g. a portfolio of short-term swap contracts) is the independent variable. Ederington (1979) shows that the estimated slope coefficient in this regression is the variance-minimizing hedge ratio. Broadly speaking, if the adjusted R^2 is greater than 80%, then a hedge ratio equal to the regression slope coefficient , would have been highly effective. The intercept term is the amount per period, on average, by which the change in value of the hedged item differs from the change in value of the hedging portfolio, and should be close to zero for effective hedging portfolios.

We present the results for contract Q1 in Table 8. The results for other quarterly and yearly contracts are not reported, because lack of liquidity (trading volume) in the replicating portfolio makes the results unreliable. For instance in the case of the Y1 contract, the replicating portfolio contains the contracts Q4, Q5 and Q6. But only in 56 days in a sample of 2180 days, that is, in less than 3% of the days, there is enough liquidity to set up the replicating portfolio. In Table 8 we may see that the intercept (a) is close to zero in most cases (excepting three cases) and the average value of the minim-variance hedge ratio is around 0.23. The adjusted R^2 ranges from 0.09 to 0.84 , with an average value of 0.48. Overall the evidence suggests that hedging quarterly contracts with monthly contracts is not a particularly effective hedging strategy in this sample. More generally, other longer-term contracts (yearly) cannot be reliably hedged using other short-term contracts (quarterly or monthly). This result is in agreement with the assumptions underlying our model with regard to the importance of the specific components of each contract, that ,in this market, constitute a significant source of unique risk[25].

5. Comparison against alternative models

In this section we compare the performance of the baseline model introduced in section 2 against two competing models, the first one is based on a one-factor spot price model and the second based on the HJM approach.

5.1 Spot Prices: One factor Model

We summarize the framework outlined in Lucia and Schwarzt (2002). Spot electricity prices P_t are characterized as

$$P_t = f(t) + X_t \tag{16}$$

where $f(t)$ is a deterministic function[26], and X_t, is a mean-reverting stochastic process with constant volatility σ and, under the natural probability measure P follow:

$$dX_t = -kX_t dt + \sigma dZ_t^P \tag{17}$$

[25] One important issue is then what kind of contracts should be used to hedge swap positions in the EEX market. The answer to this question lies beyond the realm of this paper and requires a specific paper to address it.

[26] Variables *f(t)* and *fF(t)* include constant terms, deterministic seasonal components as well as other deterministic factors such as calendar effects.

It can be shown (see Cartea and Figueroa, 2005) that under the risk-neutral probability measure Q it follows:

$$dX_t = k(\alpha^* - X_t)dt + \sigma dZ_t^Q \tag{18}$$

where dZ_t^Q are increments of standard independent Brownian motions Z_t^* the mean reversion parameters are k and X(0)=x_0, where the drift terms are

$$\alpha^* \equiv \frac{-\lambda\sigma}{k} \tag{19}$$

In this section we assume the Market Prices of Risk (MPR) of the electricity, which are λ respectively, to be constant over time. Under the risk-neutral measure the spot price P_t follows

$$P_t = f(t) + X_0 e^{-kt} + \alpha^*(1 - e^{-kt}) + \sigma \int_0^t e^{k(s-t)} dZ^Q \tag{20}$$

The distribution of P_t is Normal with mean given by :

$$E_0^Q(P_t) = f(t) + X_0 e^{-kt} + \alpha^Q(1 - e^{-kt}) \tag{21}$$

The value of any derivative security must be the expected value, under the risk-neutral measure, of its payoffs discounted to the valuation date at the risk-free rate. Assuming a constant risk-free rate *r*, the value at time zero of a forward contract on the spot price maturing at time *T* must be

$$V_0^T(P_T) = e^{-rT} E_0^*[P_T - F_0(P_0, T)] \tag{22}$$

where $F_0(P_0,T)$ is the forward price set at time zero and *T* is the time to maturity. Since the value of a forward contract must be zero when it is first entered into, we obtain a closed form expression for computing forward prices with maturity *T* as follows

$$F_0(P_{0,}T) = E_0^*(P_T) = f(T) + (P_0 - f(0))e^{-kT} + \alpha^*(1 - e^{-kT}) \tag{23}$$

The variance of the forward prices are given by

$$Var_0^T(P_T) = \frac{\sigma^2}{2k}(1 - e^{-2kT}) \tag{24}$$

These results are for forward contracts providing electricity in a single point in time (T). Given that the swap contract provides delivery of electricity during a period of time (e.g. during 31 days in January), we use (23) to generate prices during the full delivery period (e.g. we generate thirty-one forward prices in the cases of monthly contracts maturing in January and so on) , and we take the average. This average is the estimated forward price provided by the spot price model. The results of the estimation of this model and the pricing exercise with the computation of RMSE are in Appendix A. As an illustration we present the results for contracts M1, Q1 and Y1 (which are the most liquid contracts within each market segment) during 2010 and compare them against market prices. The results suggest that this model is not able to capture the basic features of swap market prices. In particular, theoretical prices are much more volatile than market prices as the graphs in Figure A2 suggest. The reason is that theoretical prices are simple functions of spot prices which are always much more volatile than swap prices. The correlation between theoretical prices and market prices is not particularly high, being (on average) 0.53, 0.41 and 0.09, for monthly, quarterly and yearly swaps respectively. The pricing errors are not independent and the theoretically-based variances do not appropriately reflect market volatility. Besides that, the residuals from the model are strongly non-normal, contradicting the basic assumptions underlying this model. In summary, this one-factor model does not capture the basic characteristics of the swap prices and therefore is unlikely to be useful for pricing or hedging purposes.

5.2 Forward Prices: HJM Model

We tried different specifications for the HJM model. We present in this section the one based on the three more liquid contracts within each segment because in this case the model gives the best results[27]. The second model is a HJM-based multi-factor stochastic process for electricity forward prices under the real-world probability measure,

[27] Additional results for other models are available on request.

$$\frac{dF_i(t,\boldsymbol{T})}{F_i(t,\boldsymbol{T})} = \sum_{k=1}^{N} \alpha_{ki}(t,\boldsymbol{T}) + \sigma_{ki}(t,\boldsymbol{T})dW_t^{ki}, \tag{25}$$

Given that we work with the most liquid contract within each market segment we propose specific parameterizations for the volatility functions in (25) as follows

$$\frac{dF_i(t,\boldsymbol{T})}{F_i(t,\boldsymbol{T})} = \alpha_i + \sigma_{1i}(t,\boldsymbol{T_i})dW_t^{1i} \tag{26}$$

$$\sigma_{1i}(t,\boldsymbol{T_i}) = e^{-k_i(\boldsymbol{T_i}-t)}\sigma_{1i} \tag{27}$$

where dW_t^{1i}, is an independent Brownian motion for all delivery periods, and $\sigma_{1i}(t,\boldsymbol{T_i})$ are volatility functions. We decide to use the parameterization (26)-(27) for the following reasons. The first question is how many factors are needed to realistically model the evolution of the forward curve. Extant empirical evidence (see Clewlow and Strickland, 2000) suggests that, in most cases, a minimum of two or three factors are needed to model the dynamics of the forward curve[28]. So we decided to start with the most parsimonious representation, that is, one factor. Regarding the specific functions to be used, equation (27) was chosen because of its analytical tractability and at the same time its ability in reflecting the well-known fact that short dated forward returns are more volatile than long dated forwards. The results of the estimation of this model are in Appendix B.

To calibrate this model we proceed as follows: We compute returns for our 3 contracts available in each case yearly contracts (Y1 to Y3), quarterly contracts (Q1 to Q3) and monthly contracts (M1 to M3). The volatility functions are recovered by eigenvector decomposition of the covariance matrix. The decomposition yields the set of independent factors that drive the evolution of the variables underlying the covariance matrix Σ. We decompose Σ into *n* (n=3) eigenvectors $\boldsymbol{v_i}$ (size 3x1) and associated eigenvalues λ_i such that Σ = RVR' where the columns of R are the eigenvectors and the principal diagonal in V contains the eigenvalues (other elements in V are zero). We only consider one eigenvalue. The first volatility function is computed by fitting the equation (27) to the data $\boldsymbol{v_1}\sqrt{\lambda_1}$.

[28] The results in Koekebakker and Ollmar (2005) suggesting that in the Nord Pool more than 10 factors are needed to explain 95% of the term structure variation are not common in other markets.

The basic statistics of the return series are in Table B1. The average return in all cases is not statistically different from zero suggesting overall null drift for the swaps, and therefore we set $\alpha_i = 0, \forall i$. The estimated standard deviation is annualized by the number of trading days (250) and varies from 13% for the contract Y3 to 32% for the M1 contract. Volatility is usually higher for the closest to maturity contracts (Samuelson effect), confirming the well-known fact that short dated forward returns tend to be more volatile than long dated forwards. Figure B1 shows (sample period from 2004 to 2012) the term structure of the volatility for each market segment. The distribution of the returns has some skewness and deviates significantly from the normal distribution, as the very high kurtosis figures on Table B1 suggest.

The eigenvalues resulting from the eigenvector decomposition tell us the importance of each eigenvector and hence the number of factors that we should include in our model. Thus the first eigenvector is the most important, explaining 87%, 89% and 94% of the total variation in the evolution of the swap curve for the monthly, quarterly and yearly contracts respectively.

Figure B2 shows the first principal component function recovered from the above procedure for each contract type. The first principal component acts to shift the forward and also acts to tilt the forward curve. The most important factor (COMP1) is positive for all maturities, but decreasing with maturity. This implies that a positive shock to the system causes all prices to shift up but by decreasing amounts, depending on the maturity. The longer the maturity the smaller the increase in prices is. The Table B2 presents the parameter estimates from the volatility function obtained in the Principal Component Analysis using equations for the entire sample 2004-2012.

Table B2 presents in Panel A the LS estimates of the parameters from the volatility functions obtained in CPA using equation $\sigma_{1i}(t, \boldsymbol{T_i}) = e^{-k_i(\boldsymbol{T}_i - t)}\sigma_{1i}$ for the full sample period 2004-2012. Panel B reports the in-sample root mean squared pricing errors (RMSEs). Errors are computed daily based on the fitted prices of forward based on estimated parameters in Panel A and compute the volatility function implied by our model and by the market Forward Prices. The RMSE are 6.05% ,19.32%,and 11.96% for monthly, quarterly and yearly contracts respectively. All of them are much higher than the ones reported in Table 7, suggesting that the SFP model does a much better job with regard to the calibration of the volatility function than the HJM model. In the case of the first volatility function parameter σ_{1i} represents the overall volatility of the forward curve whilst parameter k_i tells us how fast the forward volatility curve decreases with increasing maturity. The parameter σ_{1i} recaptures the annualized volatility averaged over all contracts of a given class. From Table B1 it is easy to see that average volatility for annual, quarterly and monthly returns is 17%, 19% and 32% respectively. These are very close to estimated parameters σ_{1i} in Table B1 which are 19%, 19% and 38% respectively. The reason of the proximity lies in the low values of the decay factor. Estimated values of this parameter k (0.15, 0.03 and 0.20) suggest a fairly slow decrease in volatility as time to maturity increases. Monthly prices present higher overall volatility and faster decrease in volatility with maturity, followed by quarterly and yearly prices. However, monthly prices present slower volatility attenuation than yearly prices. The degree of fit of the equation is very high in the case of yearly prices (99%), followed by monthly (99%) and quarterly prices (83%). To check parameter stability we have repeated the calibration exercise using different subsamples 2006-2010 and 2010-2012. A comparison of the parameters can be observed in Figure B3, which shows the stability of the parameter estimates. Overall the results suggest that parameters are reasonably stable over time.

It is fair to say that, although the model seems to fit the volatility term structure to some extent, it is completely unable to recover the skewness and kurtosis observed in the empirical distributions. The proposed model of this paper overcomes this limitation by fitting not only the volatility term structure but also the skew and kurtosis.

6. Value at Risk

In order to compare the performance of alternative models in comparison with the SFP model, we compute Value-at-Risk (VaR) at different probability levels over a one day horizon. Given a portfolio P, a time period T and a probability level Q, a loss L^* is selected, at which exists a probability Q that effective losses L, are at most L^* in period T. The loss L^* is portfolio´s VaR. Formally,

$$Prob[L^* \geq L] = Q \qquad (28)$$

and therefore VaR_Q is a quintile of asset´s returns probability density function, which defines the maximum expected loss with confidence level Q. In the following, and to be consistent with the empirical evidence in our sample, we assume that the expected one-day swap return is zero. A comparison of the VaR for standardized returns and for different probability levels Q is shown in Figure 4, for the Normal distribution and for the NIG distribution with different kurtosis parameter values. In the case of relatively low significance levels (90% and 95%), the values of VaR_Q^{Normal} tend to be higher (in absolute terms) than those of VaR_Q^{NIG}, so the latter measure will probably underestimate risk. However, in the case of high significance levels (99% and beyond) there is a very substantial difference between the two measures, because the VaR_Q^{Normal} strongly underestimates the risk in comparison with VaR_Q^{NIG}. The difference between the two measures, for a given Q, is higher; the closer to the unity is the kurtosis parameter ξ.

For the computation of the 1-day VaR for each swap contract we proceed as follows. The innovations in the spot model and in the HJM model are assumed to be normal. However, and given the limited success of the spot price model in our sample, we employ errors from the HJM model for the VaR calculations. Therefore we compute $VaR(i,T)_Q^{Normal}$ as follows

$$VaR(i,T)_Q^{Normal} = k(\sigma_{i,T}\sqrt{\Delta t}) \tag{29}$$

where the factor k depends on Q as presented in Figure 4 and $\sigma_{i,T}$ is the volatility of the innovations of the forward prices generated by means of the HJM model.

To compute the VaR in the case of the SFP model, we consider that each swap can be thought as a portfolio containing two stochastic factors and therefore its VaR should be computed using the standard VaR formula for a portfolio (Jorion, 2001). Using equation (1), we define the VaR for the swap in market segment i and maturity T as follows

$$VaR_Q^{NIG}[F_i(t,T)] = \sqrt{VaR_Q[\overline{F}_i(t)]^2 + VaR_Q[\gamma_i(t,T)]^2 + 2Cov(\overline{F}_i(t),\ \gamma_i(t,T))}$$

(30)

The cumulative distributions functions for the NIG processes driving $\bar{F}_i(t)$ and $\gamma_i(t,T)$ are computed by means of numerical simulation. The VaR for each component is computed as follows

$$VaR_Q^{NIG}[\overline{F_\iota}(t)] = k_{\xi,\chi}(\theta_{\overline{F,\iota}}\sqrt{\Delta t}) \tag{31}$$

$$VaR_Q^{NIG}[\gamma_i(t,T)] = k_{\xi,\chi}\left(\theta_{\gamma_i(T)}\sqrt{\Delta t}\right) \tag{32}$$

where the factor $k_{\xi,\chi}$ depends on Q and on the skewness and kurtosis parameters. The volatilities $\theta_{\overline{F,\iota}}$, $\theta_{\gamma_i(T)}$ are the residual standard errors obtained from equation (12) and reported in Table 5 and the covariance term is obtained from matrix $\Omega = \{\omega_{i,j}\}$ (see Table 6) and it is computed as follows

$$Cov\left(\overline{F_\iota}(t), \gamma_i(t,T)\right) = \theta_{\overline{F,\iota}} \times \theta_{\gamma_i(T)} \times \omega_{\overline{F_\iota},\gamma_i(T)}$$

Next, we compare the Failure Ratios (*FR*) of these two alternative models in our sample. We define *FR* as

$$FR = \frac{Realized\ proportion\ of\ VaR\ failures}{Expected\ proportion\ of\ VaR\ failures} = \frac{N/T}{c}$$

where 1-*c* is the confidence level, *T* is number of time periods (e.g. *T*=100 days) and a Failure appears when the realized loss (negative return) is larger than the VaR forecast. If the model producing the VaR forecasts is right in assessing the risk, we expect that *FR*≈1. On the other hand if the model tends to underestimate (overestimate) risk, then *FR* > 1 (*FR* < 1). To formally test the statistical difference from 1 of the estimated FR , we use a variation of Kupiec (1995) test, suggested by Campbell (2007). Under the assumption that the VaR under consideration is accurate, the *z*-statistic has an approximate standard normal distribution and has a known exact finite sample distribution. The *z* statistic is actually the Wald variant of the likelihood ratio statistic proposed by Kupiec (1995). One potential advantage of the Wald test over the likelihood ratio test is that it is well-defined in the case that no VaR violations occur.

Kupiec's test is not defined in this case. Moreover, the possibility that no violations occur in a relatively short period, is not trivial. The z-statistic is

$$z = \frac{\sqrt{T}(\frac{N}{T} - c)}{\sqrt{c(1-c)}} \tag{33}$$

A positive (negative) z statistic indicates that the model tends underestimate (overestimate) risk. The results are listed in Table 9. It may be seen that VaR_Q^{Normal}, calculated under the assumption of normality tends to understate risk, and this understatement is very strong for high confidence levels (99.5% and 99.99%) suggesting that tail risk is severely underestimated. On the other hand VaR_Q^{NIG} tends to mildly overestimate risk at relatively low significance levels but it is able to properly account for extreme tail risk. It is worth noting that the overestimation of risk provided by the NIG distribution is proportionally much lower than the underestimation of risk produced by the normal distribution.

One important practical implication of our results is as follows. It is well known that VaR models allow users to control risk and decide how to allocate limited resources. Financial intermediaries impose a capital charge to traders based on risk adjusted capital. This creates a natural incentive for traders to take a position only when they have strong views on markets. If they have no views they should abstain from trading. Our results suggest that the risk adjusted capital for traders in the EEX should be adjusted upwards in comparison with the standard practice based on the normality assumption.

Traders should also rationally adjust positions as risk changes (in the face of an increasingly volatile environment a sensible response is to scale down positions). Furthermore and given that VaR is also a performance evaluation tool, the evaluators of the performance of the traders should adjust their measures accordingly.

[INSERT TABLE 8]

7. Conclusions

It is important to develop models of electricity markets that are able to handle the non-Gaussian nature of electricity prices. The reason is that risk measures and derivatives prices depend on the distribution used in a model. Growing evidence suggests that swap electricity prices result from at least three driving forces: a first stochastic factor acting as an anchor of the overall level of the curve, a second factor reflecting stable seasonal components and a third factor accounting from stochastic deviations from the previous two factors, these deviation depending on the time to maturity and the length of the delivery period. Besides that, the innovations of the stochastic factors are clearly non-normal, a critical fact that should be taken properly into account. Our model takes into account all these features. With an empirical application based on EEX during the period from 2004 to 2013, we demonstrate that, ignoring the effect of these factors, leads to a biased estimation of the level and volatility of swap prices. Furthermore, failure for account for asymmetries and fat tails leads to theoretical prices that are not compatible with market prices. By accounting for all these factors, our model gains extra explanatory power, compared with a one factor model based on spot prices and with a HJM-based model. One important implication of our results is that each swap contract is exposed to a common risk factor, but also it is exposed to an idiosyncratic risk factor, and therefore cannot be perfectly hedged by taking the opposite position in a portfolio of swap contracts with different maturities. Another remarkable point is the ability our model has in capturing extreme tail risk, as suggested by the results of the VaR analysis. A practical implication of our results is that the capital charge to traders in the EEX based on risk adjusted capital should be adjusted upwards and that the evaluators of the performance of the traders should adjust their recommendations accordingly.

Looking forward, an application of our model to other electricity markets offers an interesting topic for further investigation, given the local behaviour of many electricity markets. The application of our model for the pricing of structured derivative contracts, hedging strategies, portfolio diversification, and risk management purposes represent other natural directions for further research.

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Table 1: Summary Statistics

This table reports some descriptive summary statistics for daily swap prices over the full sample period from 6/1/2004 to 12/31/2012. The sample size is 2179 observations. JB is the Jarque-Bera normality test. Prob is the p-value of this test under the null of normality.

Panel A

	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6
Mean	48.74	49.92	50.60	50.99	51.45	51.50	50.44	51.47	51.46	51.74	52.47	52.76	52.00	52.61	53.52	55.23	56.69	57.54
Median	47.88	48.85	49.20	49.17	49.13	49.66	48.75	49.60	49.98	51.79	52.41	51.84	51.94	53.70	54.59	55.30	56.15	57.30
Max	98.41	96.76	98.23	101.94	101.00	102.75	97.50	100.93	94.95	84.75	94.07	98.33	90.15	89.00	89.67	90.30	96.30	96.80
Min	26.50	26.45	28.25	29.05	30.35	30.55	28.69	31.17	31.05	30.50	30.55	31.70	33.12	33.70	34.40	36.69	37.51	38.29
Std. Dev.	12.82	12.95	13.15	13.01	12.86	12.48	12.76	12.25	10.89	10.33	11.30	11.54	9.90	9.58	9.73	10.04	11.09	11.13
Skewness	0.80	0.79	1.04	1.23	1.25	1.19	0.98	1.19	0.56	0.36	0.72	0.81	0.64	0.30	0.27	0.25	0.58	0.55
Kurtosis	3.76	3.68	4.29	5.06	5.37	5.37	4.25	5.28	3.43	3.21	4.22	4.65	4.72	4.48	4.33	3.76	4.18	4.07
JB	287.2	269.4	541.4	939.8	1072.9	1024.5	488.0	985.3	132.3	51.67	324.9	485.2	415.6	232.4	187.5	73.78	249	215.5
Prob.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Panel B

	Average M	Average Q	Average Y
Mean	50.53	51.73	54.60
Median	48.98	50.73	54.83
Max	99.85	95.09	92.04
Min	28.53	30.61	35.62
Std. Dev.	12.88	11.51	10.24

Table 2: Average Forward Price Statistics

This table reports some descriptive summary statistics for average forward prices computed as the geometric average of daily swap prices within each market segment and over the full sample period from 6/1/2004 to 12/31/2012. The sample size is 2179 observations. JB is the Jarque-Bera normality test. Prob is the p-value of this test under the null of normality.

	$\bar{F}$			$\nabla ln\bar{F}$		
	M	**Q**	**Y**	**M**	**Q**	**Y**
Mean	50.33	51.47	54.52	0.00	0.00	0.00
Median	48.63	50.52	55.19	0.00	0.00	0.00
Maximum	93.63	92.47	91.62	0.13	0.08	0.07
Minimum	30.41	32.21	35.78	-0.08	-0.07	-0.06
Std. Dev.	12.01	10.26	10.01	0.013	0.011	0.008
Skewness	1.09	0.87	0.45	0.33	0.09	0.33
Kurtosis	4.84	5.02	4.52	9.81	9.61	12.61
JB	742	644	283	4251	3973	8426
Probability	0.00	0.00	0.00	0.00	0.00	0.00

Correlation	$ln\bar{F}_M$	$ln\bar{F}_Q$	$ln\bar{F}_Y$
$ln\bar{F}_M$	1		
$ln\bar{F}_Q$	0.94	1	
$ln\bar{F}_Y$	0.77	0.91	1

Correlation	$\nabla ln\bar{F}_M$	$\nabla ln\bar{F}_Q$	$\nabla ln\bar{F}_Y$
$\nabla ln\bar{F}_M$	1		
$\nabla ln\bar{F}_Q$	0.84	1	
$\nabla ln\bar{F}_Y$	0.68	0.89	1

Table 3. Summary Statistics SFP

This table reports some descriptive summary statistics for SFPs, in levels $\gamma_i(T)$ and in first differences $\nabla\gamma_i(T)$ the full sample period from 6/1/2004 to 12/31/2012. The sample size is 2179 observations. JB is the Jarque-Bera normality test. Boldface means that the test rejects the null of normality.

$\gamma_i(T)$	**M1**	**M2**	**M3**	**M4**	**M5**	**M6**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**
Mean	-0.0374	-0.0136	-0.0009	0.0086	0.0199	0.0234	-0.0362	-0.0077	0.0054	0.0066	0.0106	0.0199	-0.0482	-0.0358	-0.0186	0.0130	0.0371	0.0525
Med	-0.0326	-0.0184	-0.0027	0.0078	0.0183	0.0207	-0.0281	-0.0036	0.0169	-0.0023	0.0112	0.0343	-0.0361	-0.0304	-0.0178	0.0050	0.0295	0.0492
Max	0.2189	0.2079	0.1762	0.1885	0.1911	0.3115	0.2565	0.1126	0.2405	0.2054	0.1567	0.2479	0.1182	0.0252	0.0207	0.0939	0.1254	0.1313
Min	-0.3101	-0.2151	-0.1529	-0.1008	-0.1803	-0.2508	-0.3279	-0.1775	-0.1194	-0.0855	-0.1359	-0.1867	-0.2444	-0.1096	-0.0579	-0.0374	-0.0366	-0.0322
SD	0.0829	0.0603	0.0435	0.0432	0.0598	0.0835	0.0977	0.0681	0.0435	0.0471	0.0670	0.0757	0.0653	0.0290	0.0126	0.0275	0.0335	0.0373
Skw	-0.3892	-0.2277	-0.0669	0.3257	0.1610	0.0314	-0.4380	-0.3211	-0.9221	0.5931	0.0730	-0.2422	-0.6680	-0.2340	-0.1232	1.0704	0.4576	0.1196
Kurt	3.5317	3.5699	3.7948	3.5744	3.3661	3.6898	3.0278	2.2645	3.6156	3.3618	2.0930	2.8358	3.1008	2.3503	3.2435	3.4859	2.5723	2.2805
JB	**80.69**	**48.31**	**58.9820**	**68.4669**	**21.5790**	**43.5528**	**69.7465**	**86.5637**	**343.206**	**139.629**	**76.6219**	**23.7429**	**162.96**	**58.197**	**10.896**	**437.51**	**92.657**	**52.197**

$\nabla\gamma_i(T)$	**M1**	**M2**	**M3**	**M4**	**M5**	**M6**	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**
Mean	-0.000	-0.000	0.0001	0.0001	0.0001	0.0005	-0.0004	-0.0001	0.0002	0.0001	0.0001	0.0002	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
Med	-0.000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0003	-0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Max	0.0758	0.0401	0.0327	0.0409	0.0265	0.0455	0.0627	0.0255	0.0295	0.0244	0.0272	0.0316	0.0215	0.0217	0.0183	0.0168	0.0204	0.0475
Min	-0.077	-0.064	-0.0226	-0.0505	-0.0339	-0.0409	-0.0432	-0.0238	-0.0201	-0.0241	-0.0327	-0.0356	-0.0400	-0.0228	-0.0153	-0.0164	-0.0224	-0.0322
SD	0.0109	0.0062	0.0049	0.0059	0.0066	0.0077	0.0072	0.0041	0.0037	0.0040	0.0047	0.0051	0.0048	0.0031	0.0025	0.0030	0.0034	0.0041
Skw	-0.064	-0.101	0.1711	-0.2426	-0.0872	0.2695	0.3203	0.0060	0.5114	0.0160	-0.0667	0.1111	-0.5028	-0.1501	0.5755	0.1611	-0.0438	0.7546
Kurt	9.8119	12.362	6.1558	8.6103	5.1604	6.6495	11.5458	7.4804	8.9609	7.0074	7.3437	6.5205	7.9422	8.4932	9.4662	6.7470	10.369	22.077
JB	**80.69**	**48.31**	**58.982**	**68.466**	**21.579**	**43.552**	**69.7465**	**86.5637**	**343.206**	**139.629**	**76.6219**	**23.7429**	**162.96**	**58.197**	**10.896**	**437.51**	**92.657**	**52.197**

Table 4. Correlations Average factors and SPFs

This table presents the correlations between the returns of the average factors $\nabla ln\overline{F}_i(t)$ (i=1 is FBM, i=2 is FBQ and i=3 is FBY) and the returns of the SPF factors $\nabla\widehat{\gamma}_i(t,T)$ (DGM1 …DGY6). The sample size is 2178 observations. Boldface indicates statistical significance at 5% level.

DG	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6	FBM	FBQ	FBY
M1	1.00																				
M2	**0.42**	1.00																			
M3	**-0.30**	**-0.09**	1.00																		
M4	**-0.56**	**-0.41**	**0.09**	1.00																	
M5	**-0.59**	**-0.52**	-0.05	**0.22**	1.00																
M6	**-0.60**	**-0.54**	**-0.14**	**0.16**	**0.29**	1.00															
Q1	**0.51**	**0.44**	**0.11**	**-0.31**	**-0.44**	**-0.52**	1.00														
Q2	0.00	-0.05	**-0.14**	-0.04	0.07	**0.10**	**0.19**	1.00													
Q3	**-0.20**	**-0.19**	0.01	**0.09**	**0.15**	**0.22**	**-0.37**	0.02	1.00												
Q4	**-0.18**	**-0.15**	0.01	**0.11**	**0.11**	**0.17**	**-0.48**	**-0.34**	0.10	1.00											
Q5	**-0.21**	**-0.14**	0.01	**0.14**	**0.18**	**0.14**	**-0.46**	**-0.45**	**-0.19**	**0.11**	1.00										
Q6	**-0.23**	**-0.20**	-0.07	**0.18**	**0.20**	**0.23**	**-0.50**	**-0.40**	**-0.12**	0.00	**0.14**	1.00									
Y1	**0.18**	**0.15**	0.07	**-0.10**	**-0.19**	**-0.19**	**0.15**	-0.01	0.02	0.05	-0.03	**-0.24**	1.00								
Y2	-0.01	0.03	0.02	0.01	-0.03	-0.01	-0.09	-0.06	0.05	0.05	**0.11**	-0.01	**0.57**	1.00							
Y3	-0.08	-0.06	-0.05	0.03	**0.11**	0.08	-0.10	0.00	0.01	-0.01	0.04	**0.11**	**-0.14**	**0.20**	1.00						
Y4	-0.08	-0.08	-0.04	0.05	0.08	**0.09**	-0.07	0.01	-0.01	0.00	0.00	0.10	**-0.49**	**-0.39**	0.05	1.00					
Y5	-0.05	-0.06	-0.03	0.01	0.06	**0.09**	0.00	0.05	-0.03	-0.06	-0.05	0.06	**-0.66**	**-0.70**	**-0.32**	**0.13**	1.00				
Y6	-0.06	-0.05	-0.01	0.05	0.07	0.04	0.00	0.00	-0.04	-0.05	-0.03	0.09	**-0.60**	**-0.68**	**-0.39**	-0.02	**0.58**	1.00			
FBM	**0.46**	**0.36**	-0.06	**-0.32**	**-0.34**	**-0.37**	**0.52**	**0.23**	**-0.13**	**-0.25**	**-0.29**	**-0.37**	**0.60**	**0.30**	**-0.14**	**-0.34**	**-0.35**	**-0.30**	1.00		
FBQ	**0.17**	**0.15**	0.02	**-0.11**	**-0.13**	**-0.18**	**0.17**	**0.11**	-0.01	-0.06	-0.08	**-0.21**	**0.69**	**0.44**	**-0.13**	**-0.42**	**-0.45**	**-0.38**	**0.84**	1.00	
FBY	0.06	0.07	-0.02	-0.04	-0.02	-0.08	-0.02	0.01	0.02	0.04	0.04	-0.06	**0.40**	**0.29**	**-0.11**	**-0.30**	**-0.26**	**-0.18**	**0.69**	**0.89**	1.00

Table 5. Estimation of SURE and MNIG Models

This table presents estimates of model (12)

$$\begin{pmatrix}\nabla ln\overline{F_i}(t)\\ \nabla\widehat{\gamma}_i(t,T)\end{pmatrix}=\begin{pmatrix}\kappa_i(\zeta_i-ln\overline{F_i}(t-1))\\ \varpi_{i,T}\widehat{\gamma}_i(t-1,T)\end{pmatrix}+\begin{pmatrix}\theta_{\overline{F_i}}\varepsilon_{\overline{F_i}}(t)\\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t)\end{pmatrix}\quad i=1(M),2(Q),3(Y)\qquad T=1,\dots,6\quad t=1,\dots,n$$

The SURE system contains 21 equations (3+6×3) and is estimated using the feasible generalized least squares (FGLS) method. To take into account residual autocorrelation a first order AR(1) term is included in all equations. In the second step we use the standardized residuals from equation (12) $\varepsilon_{\overline{F_i}}$ and $\varepsilon_{\gamma_i(T)}$ using the Urzua (1997) method as estimations of the vector ***dL****(t)*, which is MNIG. The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations. Standard errors for the parameters in the first step are heteroskedasticity- and autocorrelation consistent. Standard errors for the NIG parameters are based on generating 500 bootstrapped samples. **, or * indicates that the coefficient estimate is significant at the 1%, or 5% level, respectively. The likelihood ratio test is computed as follows LR = -2*LOGLK(N(0,1)) + 2*LOGLN(NIG(α,β,δ,μ)). for each equation and marginal distribution. Panel A contains the results for the average series factor and the estimation of their marginal NIG distributions. Panels B,C and D contain the same information for the SFP of contract within each market segment (monthly, quarterly and yearly). Panel E contains the estimation of the parameters of the MNIG distribution of the standardized and orthogonalized residuals from equation (12).

Panel A

	$\nabla ln\overline{F_M}(t)$	$\nabla ln\overline{F_Q}(t)$	$\nabla ln\overline{F_Y}(t)$
κ_i	0.0013**	0.0012**	0.0012**
ζ_i	4.0625**	4.0594**	4.0557**
$\theta_{\overline{F_i}}$	0.0132	0.0110	0.0081
α	0.6044**	0.5381**	0.6631**
β	0.0298	-0.0263	0.0671*
δ	0.6003**	0.5027**	0.6279**
μ	-0.0297	0.0246	-0.0631
χ	0.0447	-0.0457	0.0896*
ξ	0.9058**	0.9345**	0.8863**
LR test	41119**	37137**	29287**

Panel B

	$\nabla\widehat{\gamma_M}(t,1)$	$\nabla\widehat{\gamma_M}(t,2)$	$\nabla\widehat{\gamma_M}(t,3)$	$\nabla\widehat{\gamma_M}(t,4)$	$\nabla\widehat{\gamma_M}(t,5)$	$\nabla\widehat{\gamma_M}(t,6)$
$\varpi_{i,T}$	-0.0110**	-0.0142**	-0.0331**	-0.0317**	-0.0174**	-0.0108**
$\theta_{\gamma_i(T)}$	0.0109	0.0062	0.0049	0.0059	0.0065	0.0077
α	0.7106**	0.7509**	0.8458**	0.7722**	0.7633**	0.5992**
β	0.0377	0.0406	0.0388	-0.0043	-0.0006	0.0675*
δ	0.6345**	0.6549**	0.7702**	0.6793**	0.7026**	0.5688**
μ	-0.0337	-0.0355	-0.0354	0.0038	0.0006	-0.0645
χ	0.0462	0.0462	0.0369	-0.0047	-0.0007	0.1028*
ξ	0.8706**	0.8549**	0.8033**	0.8436**	0.8423**	0.9122**
LR test	31519**	37140**	23287**	14753**	17695**	18847**

Panel C

	$\widehat{\nabla\gamma_Q}(t,1)$	$\widehat{\nabla\gamma_Q}(t,2)$	$\widehat{\nabla\gamma_Q}(t,3)$	$\widehat{\nabla\gamma_Q}(t,4)$	$\widehat{\nabla\gamma_Q}(t,5)$	$\widehat{\nabla\gamma_Q}(t,6)$
$\boldsymbol{\varpi_{i,T}}$	-0.0127**	-0.0139**	-0.0229**	-0.0187**	-0.0118**	-0.0099**
$\boldsymbol{\theta_{\gamma_i(T)}}$	0.0072	0.0041	0.0036	0.004	0.0047	0.005
α	0.7268**	0.8878**	0.9172**	0.9979**	0.9157**	0.7985**
β	-0.0252	0.0339	0.0161	0.0484	0.0709*	0.031
δ	0.7002**	0.8953**	0.9151**	0.9973**	0.9154**	0.8102**
μ	0.0243	-0.0342	-0.0161	-0.0484	-0.0711	-0.0315
χ	-0.0296	0.0293	0.0132	0.0344	0.0584*	0.0315
ξ	0.8545**	0.7659**	0.7517**	0.7087**	0.7532**	0.8122**
LR test	29519**	41137**	38767**	16113**	19655**	14567**

Panel D

	$\widehat{\nabla\gamma_Y}(t,1)$	$\widehat{\nabla\gamma_Y}(t,2)$	$\widehat{\nabla\gamma_Y}(t,3)$	$\widehat{\nabla\gamma_Y}(t,4)$	$\widehat{\nabla\gamma_Y}(t,5)$	$\widehat{\nabla\gamma_Y}(t,6)$
$\boldsymbol{\varpi_{i,T}}$	-0.0064**	-0.0059**	-0.0067**	-0.0064**	-0.0067**	-0.0059**
$\boldsymbol{\theta_{\gamma_i(T)}}$	0.0048	0.003	0.0025	0.0029	0.0033	0.004
α	0.8279**	1.0391**	0.9974**	0.6247**	0.4981**	0.4438**
β	-0.0482	0.0067	0.0799*	0.0519*	0.0176	0.0275
δ	0.8136**	1.0307**	0.9737**	0.6405**	0.5006**	0.4383**
μ	0.0475	-0.0067	-0.0783	-0.0533	-0.0177	-0.0272
χ	-0.0467	0.0044	0.0572*	0.0743*	0.0334	0.0594*
ξ	0.8017**	0.6879**	0.7138**	0.8951**	0.9432**	0.9596**
LR test	37779**	48127**	35541**	14432**	16755**	14517**

Panel E

MNIG	α	β	δ	μ
M1	0.6936**	0.0108	0.6838**	0.0123
M2	0.6936**	0.0004	0.6838**	0.0023
M3	0.6936**	0.0251	0.6838**	0.0011
M4	0.6936**	-0.025	0.6838**	0.0345
M5	0.6936**	-0.008	0.6838**	0.0078
M6	0.6936**	0.0449	0.6838**	0.0052
Q1	0.6936**	-0.013	0.6838**	0.0112
Q2	0.6936**	0.0086	0.6838**	0.0100
Q3	0.6936**	0.0293	0.6838**	0.0243
Q4	0.6936**	0.0324	0.6838**	-0.034
Q5	0.6936**	0.0334	0.6838**	-0.016
Q6	0.6936**	0.039	0.6838**	-0.048
Y1	0.6936**	-0.048	0.6838**	-0.071
Y2	0.6936**	0.0051	0.6838**	0.0315
Y3	0.6936**	0.0193	0.6838**	0.0475
Y4	0.6936**	0.0337	0.6838**	0.0067
Y5	0.6936**	0.0161	0.6838**	0.0783
Y6	0.6936**	0.0113	0.6838**	0.0533
FBM	0.6936**	0.0457	0.6838**	-0.017
FBQ	0.6936**	0.0347	0.6838**	-0.027
FBY	0.6936**	0.0285	0.6838**	0.0475

Table 6. Correlations of the residuals of model (12)

This table presents the correlations of the residuals from (12) $\begin{pmatrix}\nabla ln\bar{F}_i(t)\\ \nabla\hat{\gamma}_i(t,T)\end{pmatrix} = \begin{pmatrix}\kappa_i(\zeta_i - ln\bar{F}_i(t-1))\\ \varpi_{i,T}\hat{\gamma}_i(t-1,T)\end{pmatrix} + \begin{pmatrix}\theta_{\bar{F}_i}\varepsilon_{\bar{F}_i}(t)\\ \theta_{\gamma_i(T)}\varepsilon_{\gamma_i(T)}(t)\end{pmatrix}$ *i=1(M),2(Q),3(Y)* *T=1,…,6* *t=1,…,n* The sample size is 2178 observations. Boldface indicates the correlations needed in order to compute the term structure of swap prices given by (8).

DG	M1	M2	M3	M4	M5	M6	Q1	Q2	Q3	Q4	Q5	Q6	Y1	Y2	Y3	Y4	Y5	Y6	FBM	FBQ	FBY
M1	1.00	0.42	-0.30	-0.57	-0.60	-0.60	0.51	0.00	-0.21	-0.19	-0.22	-0.24	0.18	-0.01	-0.08	-0.08	-0.05	-0.06	0.47	0.17	0.06
M2	0.42	1.00	-0.09	-0.42	-0.52	-0.54	0.44	-0.04	-0.19	-0.15	-0.14	-0.21	0.15	0.02	-0.06	-0.08	-0.06	-0.05	0.37	0.15	0.07
M3	-0.30	-0.09	1.00	0.10	-0.05	-0.14	0.11	-0.14	0.00	0.01	0.01	-0.07	0.08	0.02	-0.05	-0.04	-0.03	-0.02	-0.06	0.02	-0.02
M4	-0.57	-0.42	0.10	1.00	0.23	0.16	-0.31	-0.05	0.10	0.12	0.15	0.18	-0.10	0.01	0.03	0.05	0.02	0.05	-0.32	-0.11	-0.04
M5	-0.60	-0.52	-0.05	0.23	1.00	0.30	-0.44	0.06	0.16	0.12	0.18	0.21	-0.19	-0.01	0.11	0.08	0.05	0.07	-0.35	-0.14	-0.02
M6	-0.60	-0.54	-0.14	0.16	0.30	1.00	-0.53	0.10	0.24	0.18	0.15	0.24	-0.20	-0.01	0.08	0.10	0.09	0.04	-0.38	-0.18	-0.08
Q1	0.51	0.44	0.11	-0.31	-0.44	-0.53	1.00	0.19	-0.37	-0.49	-0.47	-0.52	0.16	-0.09	-0.11	-0.07	0.00	0.00	0.53	0.18	-0.02
Q2	0.00	-0.04	-0.14	-0.05	0.06	0.10	0.19	1.00	0.02	-0.35	-0.46	-0.41	0.00	-0.06	-0.01	0.00	0.05	0.01	0.23	0.11	0.01
Q3	-0.21	-0.19	0.00	0.10	0.16	0.24	-0.37	0.02	1.00	0.10	-0.18	-0.11	0.01	0.05	0.01	0.00	-0.02	-0.03	-0.14	-0.01	0.02
Q4	-0.19	-0.15	0.01	0.12	0.12	0.18	-0.49	-0.35	0.10	1.00	0.12	0.01	0.04	0.05	0.00	0.00	-0.06	-0.05	-0.25	-0.07	0.04
Q5	-0.22	-0.14	0.01	0.15	0.18	0.15	-0.47	-0.46	-0.18	0.12	1.00	0.16	-0.03	0.11	0.04	0.00	-0.05	-0.03	-0.29	-0.08	0.04
Q6	-0.24	-0.21	-0.07	0.18	0.21	0.24	-0.52	-0.41	-0.11	0.01	0.16	1.00	-0.25	0.00	0.12	0.11	0.06	0.09	-0.38	-0.22	-0.06
Y1	0.18	0.15	0.08	-0.10	-0.19	-0.20	0.16	0.00	0.01	0.04	-0.03	-0.25	1.00	0.56	-0.15	-0.51	-0.67	-0.61	0.60	0.69	0.40
Y2	-0.01	0.02	0.02	0.01	-0.01	-0.01	-0.09	-0.06	0.05	0.05	0.11	0.00	0.56	1.00	0.21	-0.40	-0.71	-0.68	0.31	0.45	0.30
Y3	-0.08	-0.06	-0.05	0.03	0.11	0.08	-0.11	-0.01	0.01	0.00	0.04	0.12	-0.15	0.21	1.00	0.07	-0.32	-0.39	-0.14	-0.13	-0.11
Y4	-0.08	-0.08	-0.04	0.05	0.08	0.10	-0.07	0.00	0.00	0.00	0.00	0.11	-0.51	-0.40	0.07	1.00	0.16	0.00	-0.35	-0.44	-0.31
Y5	-0.05	-0.06	-0.03	0.02	0.05	0.09	0.00	0.05	-0.02	-0.06	-0.05	0.06	-0.67	-0.71	-0.32	0.16	1.00	0.59	-0.35	-0.46	-0.27
Y6	-0.06	-0.05	-0.02	0.05	0.07	0.04	0.00	0.01	-0.03	-0.05	-0.03	0.09	-0.61	-0.68	-0.39	0.00	0.59	1.00	-0.31	-0.38	-0.19
FBM	**0.47**	**0.37**	**-0.06**	**-0.32**	**-0.35**	**-0.38**	0.53	0.23	-0.14	-0.25	-0.29	-0.38	0.60	0.31	-0.14	-0.35	-0.35	-0.31	1.00	0.84	0.69
FBQ	0.17	0.15	0.02	-0.11	-0.14	-0.18	**0.18**	**0.11**	**-0.01**	**-0.07**	**-0.08**	**-0.22**	0.69	0.45	-0.13	-0.44	-0.46	-0.38	0.84	1.00	0.89
FBY	0.06	0.07	-0.02	-0.04	-0.02	-0.08	-0.02	0.01	0.02	0.04	0.04	-0.06	**0.40**	**0.30**	**-0.11**	**-0.31**	**-0.27**	**-0.19**	0.69	0.89	1.00

Table 7: Volatility Term Structure

This table presents the market volatility of swap returns and estimated volatility using the term structure of swap prices variances. The term structure of swap prices variances is given by $\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,\iota}}\theta_{\gamma_i(T)}v_{\bar{F}_\iota,\gamma_i(T)}$; $i = 1, \dots, I$; $T = 1, \dots, N$ where $\eta_i^2 = \theta_{\overline{F,\iota}}{}^2 \times v_{\overline{F_\iota,F_\iota}}$ and $\tau_{i,T}^2 = \theta_{\gamma_i(T)}{}^2 \times v_{\gamma_i(T),\gamma_i(T)}$ We set $\varphi_i^2(t,T) = \eta_i^2 + \tau_{i,T}^2 + 2\,\theta_{\overline{F,\iota}}\theta_{\gamma_i(T)}\omega_{\bar{F}_\iota,\gamma_i(T)}$ where $\eta_i^2 = \theta_{\overline{F,\iota}}{}^2 \times \omega_{\overline{F_\iota,F_\iota}}$ and $\tau_{i,T}^2 = \theta_{\gamma_i(T)}{}^2 \times \omega_{\gamma_i(T),\gamma_i(T)}$ are the variances of the corresponding average factor and of the stochastic discount factor respectively and $\omega_{\bar{F}_\iota,\gamma_i(T)}$ is the covariance between the two factors, all of them obtained from (12).

	Market	Model	Absolute Error	Relative Error
M1	0.3264	0.3269	-0.30%	0.30%
M2	0.2613	0.2620	-0.59%	0.59%
M3	0.2190	0.2190	-0.03%	0.03%
M4	0.2002	0.2003	-0.17%	0.17%
M5	0.1999	0.1991	0.76%	0.76%
M6	0.1997	0.1988	0.94%	0.94%
Q1	0.2245	0.2248	-0.32%	0.32%
Q2	0.1925	0.1928	-0.29%	0.29%
Q3	0.1831	0.1830	0.12%	0.12%
Q4	0.1815	0.1813	0.26%	0.26%
Q5	0.1843	0.1840	0.28%	0.28%
Q6	0.1759	0.1753	0.68%	0.68%
Y1	0.1738	0.1739	-0.16%	0.16%
Y2	0.1506	0.1511	-0.72%	0.72%
Y3	0.1311	0.1312	-0.12%	0.12%
Y4	0.1237	0.1231	1.06%	1.06%
Y5	0.1264	0.1259	0.90%	0.90%
Y6	0.1336	0.1330	0.83%	0.83%
Average			**0.17%**	**0.47%**

Table 8: Hedging Effectiveness

This table presents the results of hedging effectiveness as measured by the R^2 in the regression Y= a +bX where Y is the change in the price of the Q1 swap contract and X is the change in the hedging portfolio. The regression is run each month because the composition of the hedging portfolio changes every month. For example in the case of January the regression is $\nabla Q1 = a + b(\nabla M3 + \nabla M4 + \nabla M5) + e$. **, or * indicates that the coefficient estimate is significant at the 1%, or 5% level, respectively. The rows with the caption Q1 contain the structure of the replicating portfolio for each month.

	January	**February**	**March**	**April**	**May**	**June**
Q1	(M3+M4+M5)	(M2+M3+M4)	(M1+M2+M3)	(M3+M4+M5)	(M2+M3+M4)	(M1+M2+M3)
a	0	-0.03	0.13*	0	-0.05*	0.29**
b	0.24**	0.24**	0.30**	0.30**	0.25**	0.26**
R^2	0.64	0.74	0.45	0.84	0.67	0.15
	July	**August**	**September**	**October**	**November**	**December**
Q1	(M3+M4+M5)	(M2+M3+M4)	(M1+M2+M3)	(M3+M4+M5)	(M2+M3+M4)	(M1+M2+M3)
a	-0.12**	0.01	0.09	0.02	0	-0.37**
b	0.16**	0.11**	0.23**	0.17**	0.22**	0.32**
R^2	0.53	0.31	0.09	0.47	0.68	0.15

Table 9: VaR Wald Test

This table reports the results of Wald test for daily swap prices. The sample period spans from 6/1/2004 to 12/31/2012. The sample size is 2178 observations. **, or * indicates that the coefficient estimate is significantly different from the null hypotheses at the 1%, or 5 level, respectively. VaR is computed as

$$VaR(i,T)_Q^{Normal} = k\left(\sigma_{i,T}\sqrt{\Delta t}\right)$$

in the Normal case and as

$$VaR_Q^{NIG}[F_i(t,T)] = \sqrt{VaR_Q[\overline{F}_i(t)]^2 + VaR_Q[\gamma_i(t,T)]^2 + 2Cov(\overline{F}_i(t), \quad \gamma_i(t,T))}$$

in the NIG case. Under the assumption that the VaR under consideration is accurate, the z-statistic $z = \frac{\sqrt{T}(\frac{N}{T}-c)}{\sqrt{c(1-c)}}$ has an approximate standard normal distribution. A positive (negative) *z* statistic indicates that the model underestimates (overestimates) risk.

	N(1%)	N(0.5%)	N(0.01%)	NIG(1%)	NIG(0.5%)	NIG(0.01%)
M1	2.84**	3.07**	14.53**	-1.24	-1.79	1.68
M2	3.27**	4.59**	8.10**	-2.32**	-2.39**	1.68
M3	2.84**	5.50**	10.24**	-3.18**	-3.30**	-0.47
M4	2.63**	3.98**	14.53**	-3.39**	-3.30**	-0.47
M5	3.70**	4.89**	16.67**	-3.18**	-3.30**	-0.47
M6	4.35**	6.43**	18.81**	-3.18**	-3.30**	-0.47
Q1	3.70**	6.10**	10.24**	-3.18**	-2.70**	-0.47
Q2	4.56**	6.41**	18.81**	-4.04**	-3.30**	-0.47
Q3	3.70**	6.41**	16.67**	-2.96**	-2.39**	-0.47
Q4	2.41**	3.67**	18.81**	-2.96**	-3.00**	-0.47
Q5	4.35**	6.41**	23.10**	-3.83**	-2.70**	-0.47
Q6	5.21**	7.93**	14.53**	-3.83**	-2.70**	1.68
Y1	4.35**	6.71**	16.67**	-0.17	-1.49	1.68
Y2	4.13**	6.71**	10.24**	-1.46	-2.39**	1.68
Y3	3.70**	6.10**	10.24**	-1.68	-2.70**	1.68
Y4	2.63**	3.67**	10.24**	-3.18**	-2.70**	1.68
Y5	2.63**	5.50**	20.96**	-1.89	-2.39**	-0.47
Y6	3.70**	6.10**	25.24**	-1.68	-2.09*	-0.47

Figure 1: Swap Price Series

Figure 2: Average Forward Prices

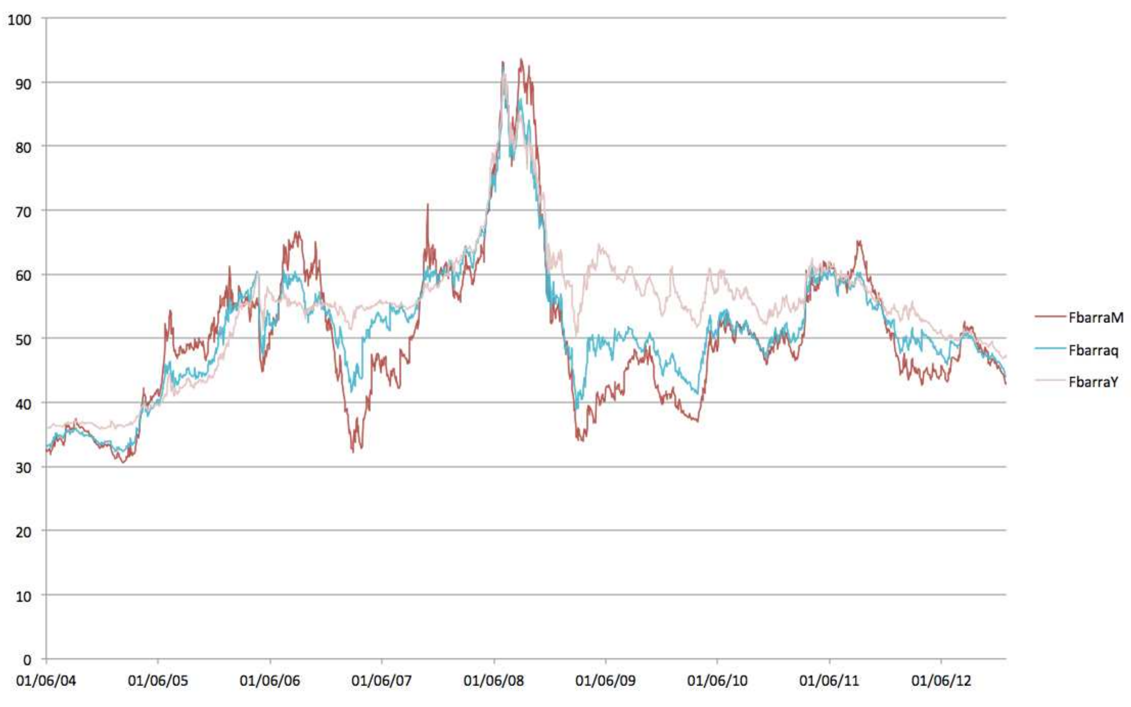


Figure 3: Monthly Seasonal Components

The figure presents the estimated seasonal components $\widehat{s_i}(K) = \frac{1}{n}\sum_{T_{k\in A_{(i)}}}\sum_{t=1}^{n}(\ln F_i(t,T_k) - ln\bar{F}\,(t))$, where $A_{(i)}$ are the sets of available maturities at time t for contracts with delivery period i=1,...,I. And the index *K* refers to each month (k=1…12). Summary statistics for these seasonal components are below the plot. The last 3 columns present an interval of confidence for them and the t-test of the null hypothesis of zero mean.

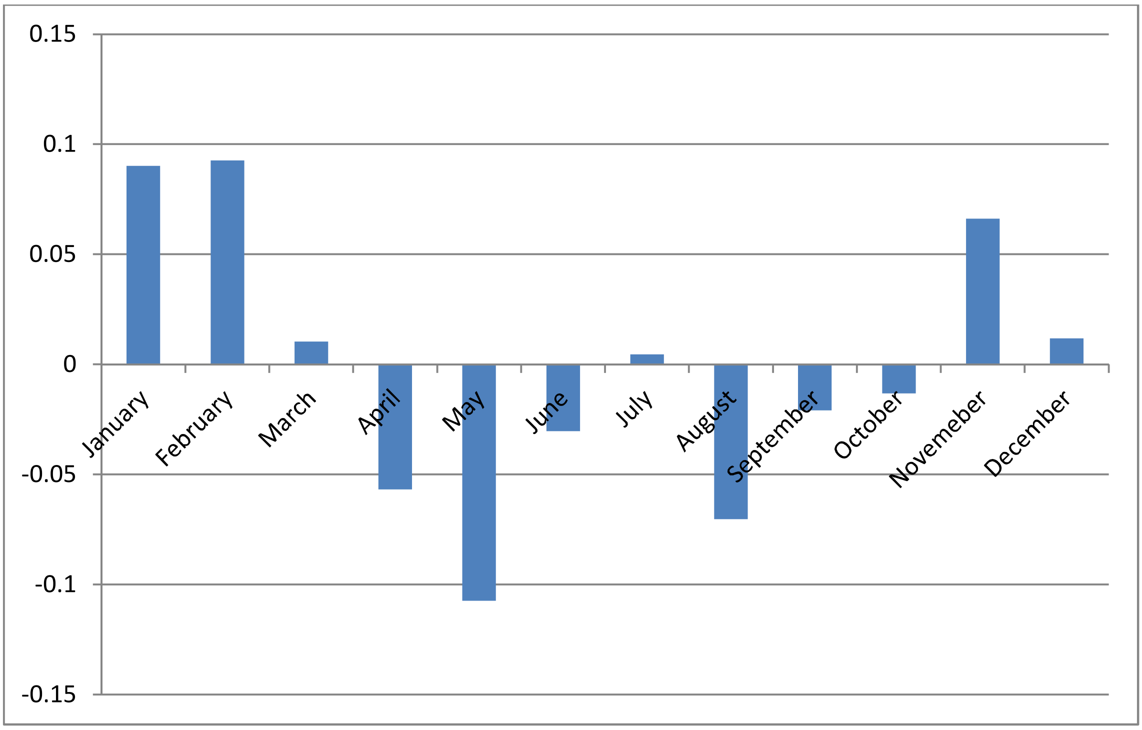
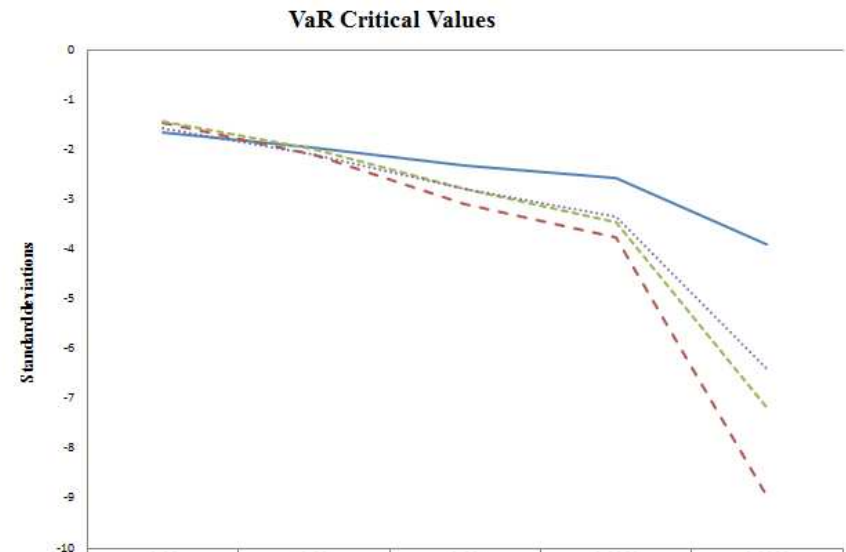


Figure 4: Critical values of Value-at-Risk for Different Levels of Significance (Q): Standardized Normal and NIG distributions



	0,95	0,98	0,99	0,9950	0,9999
k Normal	-1,65	-1,96	-2,33	-2,58	-3,89
k NIG (0,90)	-1,45	-2,09	-3,09	-3,77	-8,94
k NIG (0,86)	-1,44	-1,99	-2,8	-3,46	-7,18
k NIG (0,73)	-1,58	-2,09	-2,79	-3,34	-6,39

Appendix A: Spot Prices - One factor Model

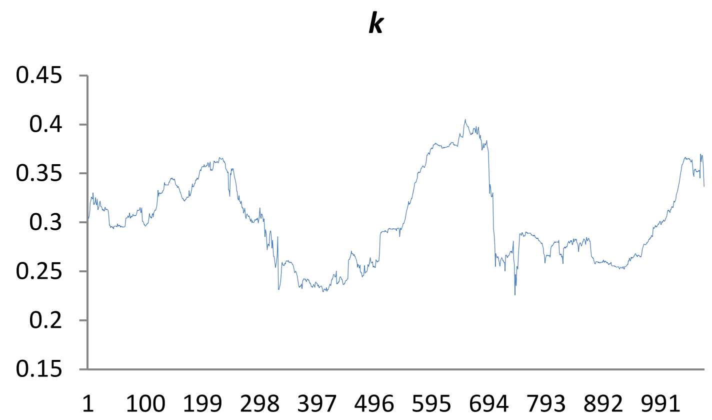
Table A1: Estimation of the One Factor Model

This table reports the results of regressions (16) and (17). The dependent variable is the average daily EEX spot price (EEX - Phelix Base Hr.01-24 E/Mwh). Our database spans from February 2, 2009 to December, 31 2012. Explanatory variables include day of the week dummies as well as the NEG variable which is a dummy variable taking into account negative electricity prices. It is equal to 1 if the price is negative (4 Oct 2009, 26 Dec 2009, 25 Dec 2012 y 26 Dec 2012) and it is zero otherwise. We estimate the coefficients by means of a regression robust to heteroskedasticity, and serial autocorrelation. The results presented correspond to the estimated coefficient, standard errors and t-statistics. The symbol * and ** denotes that the variable is significant at 5% and 1%, respectively.

	Coeff.	s.e.	t-stat
NEG	-79.86**	12.91	-6.18
@WEEKDAY=1	46.25**	0.77	60.09
@WEEKDAY=2	48.06**	0.64	75.30
@WEEKDAY=3	48.14**	0.67	71.49
@WEEKDAY=4	47.48**	0.66	71.76
@WEEKDAY=5	46.22**	0.71	65.28
@WEEKDAY=6	39.84**	0.60	65.99
@WEEKDAY=7	33.60**	0.68	49.21
$\mathbf{1-\hat{k}}$	0.77**	0.03	29.70
$\hat{k}$	0.23**	0.025	8.79
$\hat{\sigma}$	5.86		
R^2	58.7		

Figure A1: Parameter Stability

The figure depicts the recursive estimation of the parameters (mean reversion k and volatility σ) for the Spot Series 7 days EEX - Phelix Base Hr.01-24 E/Mwh. We use a 365-day rolling window. There are 1066 estimates. The last window is: 2/01/12-31/12/12. Average values are k = 0.23 and σ = 5.86.



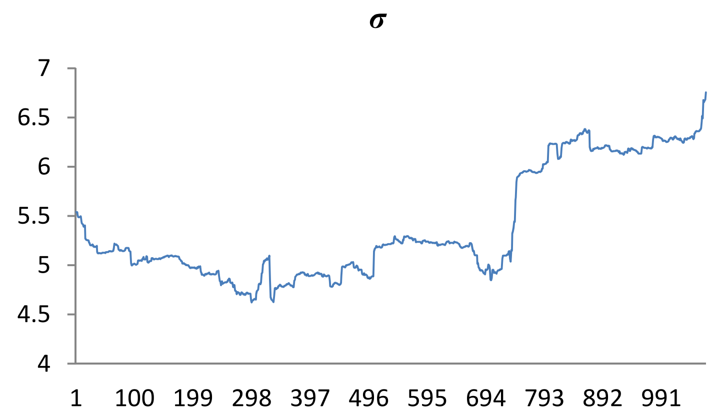
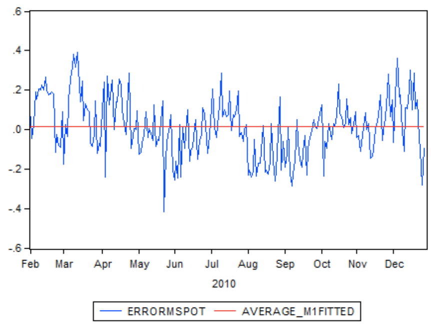


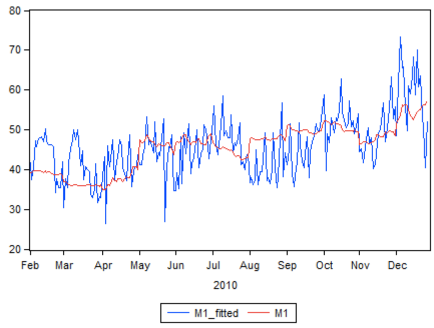
Figure A2: Forward Prices, Fitted Values and Errors in the Spot Model

Using the parameters estimated in Table B1 we compute forward prices we obtain a closed form expression for computing forward prices with maturity T using equation (23) as follows $F_0(P_{0,}T) = \overline{f(T)} + (P_0 - f(0))e^{-\hat{k}T} + \hat{\alpha}^*(1 - e^{-\hat{k}T})$. We then generate theoretical prices using the previous equation for all contracts (denoted M1_fitted, Q1_fitted and Y1_fitted). As an illustration we present the results for M+1, Q+1 and Y+1 denoted M1, Q1 and Y1 (which are the most liquid contracts within each market segment) and compare them against market prices during 2010. The results for the other contracts and time periods are available on request.

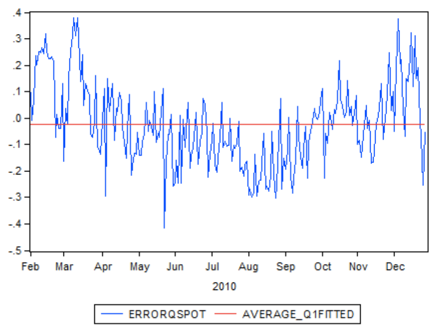
Error M1



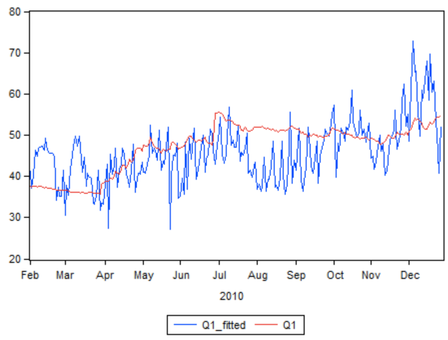
M1 Fitted and M1



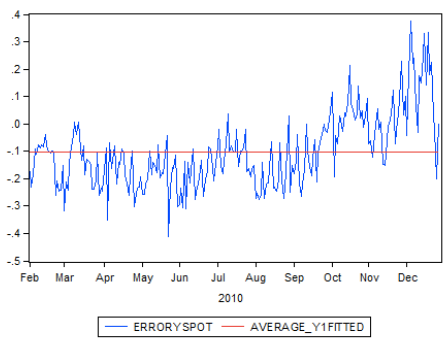
Error Q1



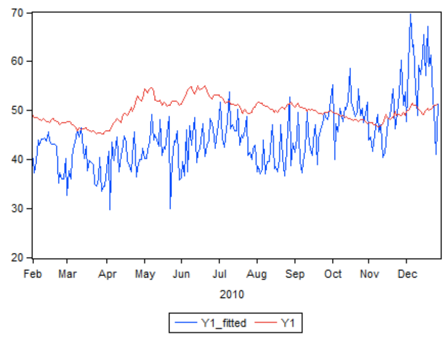
Q1 Fitted and Q1



Error Y1



Y1 Fitted and Y1



Appendix B: HJM Model

Table B1: Descriptive Statistics of Returns

The table shows descriptive statistics of returns (1-day changes in the natural logs of forward prices) and the sample covariance matrix of these returns. We study three contracts for each market segment (yearly, quarterly and monthly), contracts M1 to M3, Q1 to Q3 and Y1 to Y3 from 6/1/2004 to 12/31/2012. The "Std. Dev." column reports the standard deviation of the series in annual terms. The nine series are corrected of the rolling effect by means of intervention analysis. p-val is the p-value for the test of zero mean.

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	p-val
M1	-0.0010	-0.0006	0.1627	-0.1461	0.3289	0.1751	9.9994	0.08
M2	-0.0005	0.0000	0.1489	-0.1634	0.2688	-0.1596	14.1524	0.133
M3	-0.0004	0.0000	0.1261	-0.2391	0.2388	-1.8369	37.4440	0.195
Q1	-0.0002	0.0000	0.1091	-0.0615	0.1929	0.3261	9.8012	0.511
Q2	0.0000	0.0000	0.0994	-0.1611	0.1913	-0.9052	24.6395	0.985
Q3	0.0000	0.0000	0.0991	-0.0738	0.1818	0.1359	10.5855	0.984
Y1	0.0000	0.0000	0.0884	-0.0705	0.1739	0.0084	9.4239	0.962
Y2	0.0001	0.0000	0.0699	-0.0634	0.1502	0.1625	10.0232	0.595
Y3	0.0000	0.0000	0.0732	-0.0643	0.1312	0.5159	14.2482	0.998

Table B2: Parameters Estimation and RMSE

The table presents in Panel A the LS estimates of the parameters from the volatility functions obtained in CPA using equation $\sigma_{1i}(t,\boldsymbol{T_i}) = e^{-k_i(\boldsymbol{T_i}-t)}\sigma_{1i}$ for the full sample period 2004-2012. t- statistics are presented in parenthesis. Panel B reports in sample root mean squared pricing errors (RMSEs). Errors are computed daily based on the fitted prices of forward based on estimated parameters in Panel A and compute the volatility function implied by our model and by the market Forward Prices.

	Yearly Contracts	Quarterly Contracts	Monthly Contracts
Panel A			
$\boldsymbol{\sigma_1}$	0.0125	0.012	0.024
	(37.34)	(34.75)	(67.90)
K	-0.154	-0.0302	-0.2015
	(11.32)	(2.23)	(26.09)
$\mathbf{R^2}$	0.992	0.834	0.998
Panel B			
RMSE	6.05%	19.32%	11.96%

Figure B1: Volatility Functions

The Figure shows the first principal component functions recovered from the above procedure for each contract type (M1, M2, M3; Q1, Q2, Q3 and Y1,Y2, Y3). Sample period 2004-2012

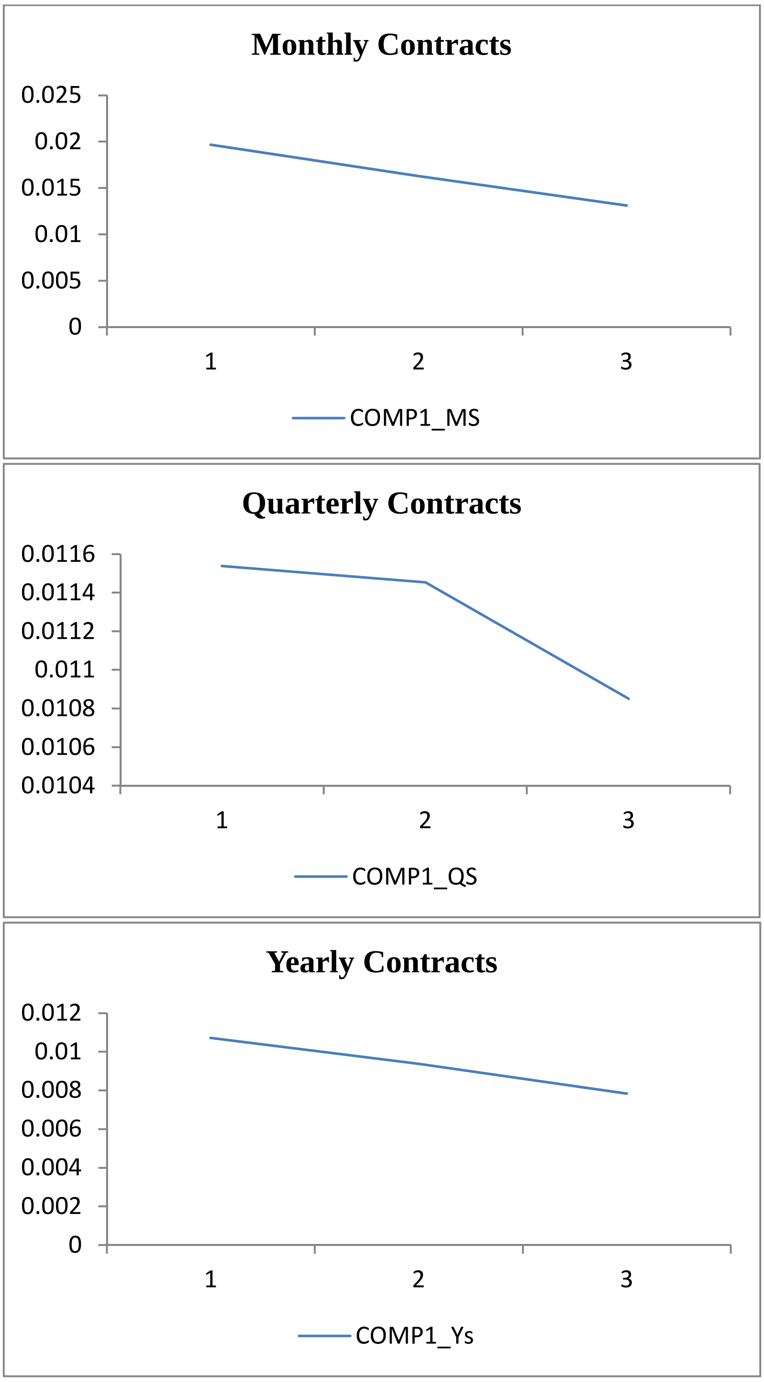
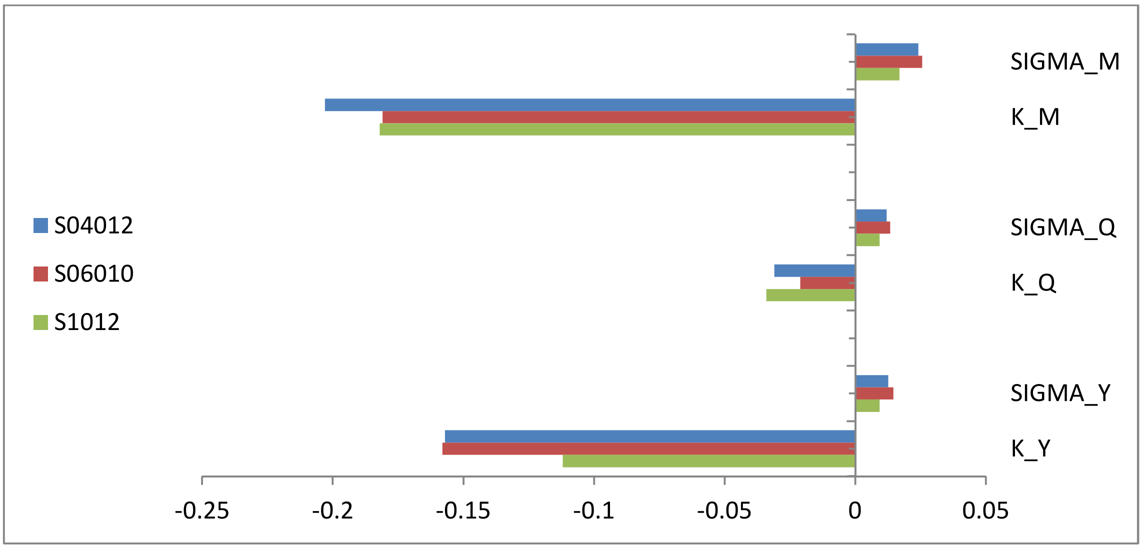


Figure B2: Stability of the parameters by subsamples

To check parameter stability we have repeated the calibration exercise using different subsamples 2006-2010 and 2010-2012. The plot presents a comparison of the parameters.



	S04012	S06010	S1012
SIGMA_M	0.0242	0.0257	0.0169
K_M	-0.203	-0.181	-0.182
SIGMA_Q	0.012	0.0134	0.0094
K_Q	-0.031	-0.021	-0.034
SIGMA_Y	0.0126	0.0146	0.0093
K_Y	-0.157	-0.158	-0.112