Arbitrage violations in currency markets

by

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Abstract

In this paper we show that the existence of arbitrage of any order, involving any number of currencies, can be treated within a simple general framework. This general type of arbitrage can be detected by using a simple linear program, and the optimal strategy (the one providing the highest riskless profit by monetary unit traded) can be computed by using a simple mixed-linear program. Using tradable quotes from EBS over the over the period 1997-2007 we document several short-lived arbitrage involving up to five currencies. The size of the arbitrage opportunities may be economically significant and is unlikely due to transaction costs, and durations are often high enough to exploit these arbitrages. Furthermore, we find that the average size of arbitrages has decreased over time, but there are still significant arbitrage opportunities toward the end of our dataset.

Key Words and Phrases: Market Microstructure; Arbitrage

1 Introduction

The foreign exchange (FX) market is the world biggest financial market with a daily trading volume of USD 5.3 trillion of which spot trading accounts for USD 2.0 trillion (B.I.S, 2013). The market is open 24 hours a day, seven days a week and is considered to be highly liquid for most business hours. It is reasonable to assume that such markets would not offer risk-free arbitrage opportunities when taking into account transactions costs.

In FX markets, three types of arbitrage opportunities may exist. One type of arbitrage is violation of covered interest rate parity (CIP), which predicts that net returns on an investment that borrows at home and lends abroad, or vice versa, should be zero if exchange rate risk is hedged through a forward contract (Akram et al., 2008 and Fong et al., 2009). A second type of arbitrage exploits the occurrence of negative spreads (Ito et al., 2012). This paper is concerned with a third type of arbitrage where arbitrage opportunities may exist if the exchange rate for a currency pair is out of line with other currency pairs (Foucault et al., 2013 and Kozhan and Tham, 2012). The current literature on this type of arbitrage opportunities has been entirely restricted to triangular arbitrage opportunities. For instance, a triangular arbitrage opportunity may involve simultaneous trading in JPY/USD, USD/EUR and JPY/EUR. This paper contributes to the literature by allowing for arbitrage opportunities of higher order, e.g. including arbitrage through four or five currencies.

We show that the existence of arbitrage of any order, involving any number of currencies, can be treated within a simple general framework. This general type of arbitrage can be detected by using a simple linear program, and the optimal strategy (the one providing the highest riskless profit by monetary unit traded) can be computed by using a simple mixed-linear program. This novel approach is formalized in the theory section of this paper.

The empirical section applies the theoretical results using a dataset containing tradeable best bid-ask quotes over the period 1997-2007 obtained from the EBS platform which represents the biggest interdealer trading venue for major currencies. Our results document the existence of several short-lived arbitrage involving up to five currencies. The size of the arbitrage opportunities may be economically significant and is unlikely due to transaction costs, and durations are often high enough to exploit these arbitrages. Our approach in this empirical effort is not to present the existence of arbitrage as opportunities to obtain riskless profits but as a way to quantify the economic impact of the deviations of real-life currency markets (fees, volume constraints, and inability to trade any amount, etc.) from their frictionless counterparts.

Our dataset covers major changes in market structure such as the introduction of computerized trading in 2003 and the Professionl Trading Community introduced in 2004 allowing e.g. hedge funds to trade through EBS. We find that the average size of arbitrages has decreased over time, but there are still significant arbitrage opportunities toward the end of our dataset. Average durations have, however, not changed much, even after the introduction of computerized trading. Our empirical findings may be consistent with Grossmann-Stiglitz suggesting that small arbitrage opportunities provide incentives to watch them, and market participants thus restore arbitrage-free prices (Grossman and Stiglitz, 1976, 1980). Alternative explanations for small arbitrage opportunities may be execution risk, that is, you do not know ex ante whether you will be able to execute all trades involved in an arbitrage (Kozhan and Tham, 2012). Another explanation may be residual risk which is relevant since you can only trade in units of currency on EBS. This means that you will not end with a zero inventory when exploiting misaligned exchange rates (Kozhan and Tham, 2012). Although we now have other trading venues that allow trading in smaller units than one million of base currency, residual risk may still be relevant. Finally, settlement risk (also known as Herstatt risk[2]) may also explain arbitrage opportuni-

[2] One form of settlement risk is foreign exchange settlement risk or cross-currency settlement risk, sometimes called Herstatt risk after the German bank that made a famous example of the risk. On 26th June 1974, the bank's license was withdrawn by German regulators at the end of the banking day (4:30pm local time) because of a lack of income and capital to cover liabilities that were due. But some banks had undertaken foreign exchange transactions with Herstatt and had already paid Deutsche Mark to the bank during the day, believing they would receive US dollars later the

ties. Although credit risk is minor in interdealer fx trading, it may still be relevant explaining small profits. However, we would expect credit risk to be far more important e.g. in explaining CIP arbitrage since this involves credit risk over a longer time period.

The remainder of the paper is organized as follows. Section 2 provides a literature review. Our theoretical contributions are presented in Section 3. Section 4 presents our data set and explain some important institutional features of the market, while results are discussed in Section 5. Section 6 concludes.

2 Related literature

Arbitrage describes the ability to achieve riskless profits with no capital outlay and no risk. Arbitrage opportunities may arise in different markets and across assets that can be designed to be perfect substitutes. In a real world setting, however, arbitrage may not be entirely risk free due to e.g. credit risk.

We can separate between three types of arbitrages in FX markets. First, and

same day in the US from Herstatt's US nostro. But after 4:30 pm in Germany and 10:30 am in New York, Herstatt stopped all dollar payments to counterparties, leaving the counterparties unable to collect their payment. The closing of Drexel Burnham Lambert in 1990 did not cause similar problems because the Bank of England had set up a special scheme which ensured that payments were completed. Barings in 1995 resulted in minor losses for counterparties in the foreign exchange market because of a specific complexity in the ECU clearing system.

may be the most obvious type of arbitrage, is when there is a negative spread. This means that the seller's ask price is lower than the buyer's bid price. In this case, arbitrage profits can be generated by simultaneously hitting seller's ask price to buy the currency and the buyer's bid price to sell the currency. Institutional details explain why such arbitrage opportunities may emerge (Ito, 2012). When a bank joins the network, the bank must open credit lines with a certain number of counterparty banks. A financially weak bank may only be able to maintain credit lines with few counterparty banks. Such a bank may not be able to hit the best bid or ask due to the lack of credit lines. The weak bank may thus post a quote (e.g. bid) that show a negative spread. Neither the weak bank nor the bank that provide the other quote (ask) will be able to exploit the opportunity due to the lack of credit lines. However, other banks that have credit lines with both banks may exploit the opportunity. Ito et al (2012) show, using tradable quotes from EBS, that negative spreads may occur. However, the probability of observing negative spreads have declined significantly over time. This is most likely due to changes in market structure that will be explained when discussing our data from EBS. Based on the discussion above, this type of arbitrage may not be entirely risk free.

A second type of arbitrage exploits violations of Covered Interest Rate Parity (CIP). Recent studies use high frequency data to explore deviations from CIP (Akram et al., 2008 and Fong et al., 2010). Akram et al. (2008) use tradable quotes from

Reuters Dealing 3000 for FX spot (USD/EUR, USD/GBP and JPY/USD). For swaps and euro-currency deposits they use indicative quotes from Reuters Monitor. They find evidence of economically significant arbitrage opportunities. Although the arbitrage opportunities are short-lived, durations are often sufficiently large such that arbitrages may be exploited. This study assumes that it is possible to trade on indicative quotes for swaps and euro-currency deposits. It is not obvious that this will be possible. Fong et al. (2010) use tradable quotes for all input needed in CIP for HKD/USD utilizing data on tradable best bid and ask for FX spot, outright forward rate and all best bid and ask quotes for the HKD- and USD-denominated deposit rates. They still find arbitrage opportunities when using tradable quotes, however, market liquidity and credit risk explain a large part of the CIP arbitrage violations.

The third type of arbitrage exploits opportunities when an exchange rate for a currency pair is out of line with other currency pairs. Compared with other forms of arbitrage, also including the two mentioned above, involves less risk. For instance, settlement risk (credit risk) is close to zero and liquidity is high for most trading hours in major currencies. Existing studies (e.g. Marshall et al., 2008, Fenn et al., 2008, Kozhan and Tham, 2012, Lyons and Moore, 2008 and Ito et al., 2012) analyze triangular arbitrages, although there may also be arbitrage opportunities involving more than three currencies. Marshall et al. 2008 use tradable quotes obtained from EBS from 2005. They find significant arbitrage opportunities, and that these opportunities are inversely related with market liquidity even after accounting for higher spreads. Fenn et al. (2008) use similar data for October 2005 obtained from HSBC bank and obtain similar results as Marshall et al. (2008). Their results suggest that average duration of arbitrage opportunities are roughly two seconds, while median duration is one second. Similar to Marshall et al. (2008) they document a larger number of arbitrage opportunities with shorter durations during the more liquid hours.

Focault et al. (2012) document arbitrage opportunities consistent with findings in e.g. Fenn et al. (2008) and Marshall et al. (2008) using tradable quote data and transaction data from Reuters Dealing 3000 for 2003 and 2004. They suggest that arbitrage may also have negative effects on liquidity because prices of related securities sometimes do not adjust to new information at the same speed. Exploitation of these opportunities generate adverse selection for liquidity providers who adjust their quotes relatively slowly. High frequency trading may increase these costs.

Koshan and Tham (2012) investigate the role of execution risk in high-frequency trading through arbitrage strategies. Using similar data as Focault et al. from 2003 and 2004, they show that rational agents face uncertainty about completing their arbitrage strategy. Arbitrage is thus limited even in markets with perfect substitutes and convertibility. They still find, however, that even after accounting for fees, bid-ask spread and latency cost (that is, may not be able to complete the strategy), triangular arbitrage opportunities remain and are not exploited instantly. The authors suggest that a possible economic reason for this is execution risk.

Chaboud et al. (2013) study algorithmic trading strategies in FX markets. Their results suggest that one type of algorithmic trading models are designed to exploit arbitrage opportunities, and may thus explain the reduction in triangular arbitrage profit over time. Chaboud et al. (2013) also suggest that algorithmic models providing market making by submitting limit orders may be constructed in such a way that arbitrage opportunities will be avoided. Furthermore, they find that algorithmic trading improves informational efficiency, but at the same time increases adverse selection costs to slower traders as suggested by e.g. Foucault et al. (2012).

3 Notation and Preliminaries

Consider a basket of N currencies and let $r_{ij}^a, r_{ij}^b > 0$ denote the number of monetary units of currency j that are paid or received, respectively, in exchange of one monetary unit of currency i, for any $i, j \in \underline{N}$ where $\underline{N} \equiv \{1, 2, \ldots, N\}$. It brings a great deal of simplicity to work with the logarithms of the bid and ask exchange rate quotes. In order to do so, define the values

$$l_{ij}^a \equiv \ln r_{ij}^a, \qquad l_{ij}^b \equiv \ln r_{ij}^b \qquad \forall i, j \in \underline{N}$$

and consider the matrices $L^a$and L^b with typical elements l^a_{ij} and l^b_{ij}, respectively.[3] We will naturally assume that

$$l^a_{ii} = l^b_{ii} = 0, \qquad \forall i \in \underline{N} \tag{1}$$

and that[4]

$$l^a_{ij} = -l^b_{ji}, \qquad \forall i, j \in \underline{N}. \tag{2}$$

It will also be convenient to define the vectors

$$\mathbf{l}^a \equiv vec\left(L^a\right) \text{ and } \mathbf{l}^b \equiv vec\left(L^b\right).$$

Consider also any summation of the form

$$l^b_{ik} + l^b_{kl} + \cdots + l^b_{pq} + l^b_{qj} \tag{3}$$

where for any two consecutive log exchange rates in the sequence, the second subindex of the first one is equal to the first subindex of the second one, and where neither the starting currency nor the finishing currency can again apper as intermediate

[3]Initially, we will be assuming that there are available bid and ask quotes for all currency pairs. Of course, this is not necessary the case in reality. Allowing for this possiblity can easily be done as we will show later, but this would severely complicate our notation. For similar reasons, we will keep in our analysis all exchange rates that exchange a particular currency into itself although their use is obviously unnecessary.

[4]Note that $l^a_{ij} = -l^b_{ji}$ is equivalent to $r^a_{ij} = 1/r^b_{ji}$ which is obviously the case in real life currency markets.

currencies in the sequence. In the sequel, we will refer to any sequence of subindex pairs which follows such pattern as *conformable.* Note that the summation above represents the number of log-units of currency j that can be obtained in exchange of one log-unit of currency i via trading some intermediate currencies $k, l, \ldots, p, q$. Obviously, similar arguments apply when the summation involves ask quotes.

Since we will be interested in expressing summations like (3) via a dot product, let X be any NxN-dimensional matrix with typical element x_{ij}, which can only take value zero or one. and consider its vectorization

$$\mathbf{x} \equiv vec\,(X)\,.$$

Also, consider any such N^2-dimensional vector of binary variables with $x_{ii} = 0\ \forall i \in \underline{N}$ and denote their set by $\underline{X}$.[5] Clearly, any conformable sequence can be expressed via a vector $\mathbf{x}$ in $\underline{X}$ whose x_{ij} element is equal to one if the subindex pair ij is in the sequence and zero otherwise. Hence, for any $i, j \in \underline{N}$, let $\underline{C}_{ij} \subset \underline{X}$ denote the set of all such vectors associated to conformable sequences which exhibit i as the first element of the first subindex pair and j as the second element of the last subindex pair. There is thus a vector $\mathbf{x}$ in $\underline{C}_{ij}$ such that the dot product

$$\mathbf{x}'\mathbf{l}^b$$

[5] We ignore the components x_{ii} even though we retain them in our definitions of our vectors x for the same reason that we also consider the trivial exchange rates l_{ii}. Doing this alleviates the burden of our notation.

is equal to (3).

It is important to note that we will follow the convention that for all $i, j \in \underline{N}$, $i \neq j$, the set $\underline{C}_{ij}$ also contains the trivial vectors $\mathbf{x}$ with$x_{ij} = 1$ and zero otherwise, such that

$$\mathbf{x}'\mathbf{l}^b = l_{ij}^b \text{ and } \mathbf{x}'\mathbf{l}^a = l_{ij}^a.$$

This convention guarantees that our definition of absence of arbitrage below excludes negative bid-ask spreads.

Definition 1 *There is no arbitrage in the currency market if*

$$\mathbf{x}'\mathbf{l}^a \geq l_{ij}^b, \qquad \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N}, \tag{4}$$

and

$$\mathbf{x}'\mathbf{l}^b \leq l_{ij}^a, \qquad \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N}. \tag{5}$$

Finally, we will use the term *recombining* to refer to conformable sequences associated to vectors in $\underline{C}_{ii}$ for any i in $\underline{N}$. We will also denote the set of all these sequences as $\underline{R}$, that is,

$$\underline{R} \equiv \left\{ \underline{C}_{11}, \ldots, \underline{C}_{NN} \right\}.$$

Since neither the starting currency nor the finishing currency of a conformable sequence is allowed to appear as an intermediate currency, our recombining sequences do not contain subsplits by definition.

4 Theoretical Results

We turn now to present some useful characterizations of arbitrage.

Theorem 2 *The following statements are equivalent:*

1. *There is no arbitrage in the currency market.*
2. $\mathbf{x}'\mathbf{l}^b \leq l_{ij}^a, \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N}$.
3. $\mathbf{x}'\mathbf{l}^a \geq l_{ij}^b, \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N}$.
4. $\mathbf{x}'\mathbf{l}^b \leq 0, \forall \mathbf{x} \in \underline{R}$.
5. $\mathbf{x}'\mathbf{l}^a \geq 0, \forall \mathbf{x} \in \underline{R}$.

Proof. We will only show that (2) $\Longrightarrow$ (1) and that (4) $\Longrightarrow$ (2). The remaining implications are either trivial or follow identical arguments.

(2) $\Longrightarrow$ (1) Note that

$$\mathbf{x}'\mathbf{l}^b \leq l_{ij}^a, \forall \mathbf{x} \in \underline{C}_{ij},$$

implies that

$$-\mathbf{x}'\mathbf{l}^b \geq -l_{ij}^a, \forall \mathbf{x} \in \underline{C}_{ij},$$

which together with (2) gives

$$\mathbf{x}\mathbf{l}^a \geq l_{ji}^b, \forall \mathbf{x} \in \underline{C}_{ji}$$

and the desired result follows by replicating this argument $\forall i, j \in \underline{N}$.

(4) $\Longrightarrow$ (2) Assume that

$$\mathbf{x}'\mathbf{l}^b \leq 0, \forall \mathbf{x} \in \underline{R}.$$

It is easy to see that the above implies

$$\mathbf{x}'\mathbf{l}^b + l_{ji}^b \leq 0, \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N}.,$$

which together with (2) gives

$$\mathbf{x}'\mathbf{l}^b - l_{ij}^a \leq 0, \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N},$$

or equivalently,

$$\mathbf{x}'\mathbf{l}^b \leq l_{ij}^a, \forall \mathbf{x} \in \underline{C}_{ij}, \forall i, j \in \underline{N}.$$

■

Clearly, in order to establish the absence of arbitrage, we can choose any of the characterizations presented in the above theorem. In the sequel, we will use (4), that is, the absense of *bid* recombining summation sequences greater than zero. In this task, it will be convenient to define the value

$$\theta \equiv \max_{\mathbf{x} \in \underline{R}} \mathbf{x}'\mathbf{l}^b. \tag{6}$$

Hence, θ gives the maximum value of all bid recombining summation sequences. It is thus trivial to show the following.

Theorem 3 *There is arbitrage in the currency market if and only if* $\theta > 0$.

The maximization defined in (6) also gives a measure of the magnitude of arbitrage violations. Since the logarithmic function is strictly monotonic, it is clear that

$$exp\left(\theta\right) - 1$$

gives the maximum value of all recombining *product* sequences. Thus, the expression above gives the maximum arbitrage profit that can be obtained per currency unit traded. The currency in question is any of the ones involved in the recombining sequence.[6]

We turn now to show how the value θ by means of simple zero-one linear programing. Since the objective function in the definition of θ is linear on the binary variables in $\mathbf{x}$, the challenge lies in finding a way to express the feasible set with a set of linear constraints. Note that if a currency i is part of a recombining sequence, it

[6] Note that a recombining product sequence such for example

$$r_{iq}r_{qj}r_{ji},$$

can also be written as

$$r_{qj}r_{ji}r_{iq}.$$

Thus, the starting and finishing currency of the whole loop can be any of the ones involved in the sequence. As we will see later, our analysis can be easily extended in order to find maximum arbitrage profits that impose the presence of a given currency or quote.

must be the case that the sequence contains one and only one subindex pair whose first subindex is i and one and only one subindex pair whose second subindex is also i. These conditions can be written by imposing that the elements of $\mathbf{x}$ satisfy

$$\sum_{i=1, i \neq j}^{N} x_{ij} = \sum_{i=1, i \neq j}^{N} x_{ji} \leq 1, \; \forall i \in \underline{N}. \tag{7}$$

For most practical purposes, this set of constraints is enough if the goal is to detect riskless arbitrage profits. However, our formal definition of a recombining sequence precludes the presence of subsplits something that the above set of constraints does not. In other words, a vector $\mathbf{x}$ with components $x_{12} = x_{21} = x_{34} = x_{45} = x_{53} = 1$ and zero otherwise, satisfies (7) and yet it is not an element of $\underline{R}$. Of course, this particular vector contains two nonoverlapping recombining sequences. In other words, the two vectors in $\underline{X}$ whose only nonzero components are x_{12} and x_{21} and x_{34}, x_{45} and x_{53}

are both in $\underline{R}$. Futhermore, their dot product with the vector $\mathbf{l}^b$ must be positive in both cases if the above vector $\mathbf{x}$ which contains them happens to maximize $\mathbf{x}'\mathbf{l}^b$ over the set $\underline{X}$ and $\mathbf{x}'\mathbf{l}^b$ is positive. Hence, if we define

$$\begin{aligned} \overline{\theta} \quad \equiv \quad & \max_{\mathbf{x} \in \underline{X}} \mathbf{x}'\mathbf{l}^b \\ & \text{subject to (7)} \end{aligned} \tag{8}$$

we can conclude the following.

Theorem 4 *There is arbitrage in the currency market if and only if* $\bar{\theta} > 0$.

Despite of this result, the value $\bar{\theta}$ satisfies $\bar{\theta} \geq \theta$ and it is not necessarily equal to θ. However, in the absence of negative spreads but in the presence of arbitrage, most practical cases will deliver solutions of the above program whose optimal vector is in $\underline{R}$ and which therefore, satisfy $\bar{\theta} = \theta$. We will get back to this discussion later. For that purpose, it will be easier to first introduce a particular case of arbitrage.

4.1 S-order arbitrage

All attention on the arbitrage literature has been devoted to the existence of negative spreads and triangular arbitrage. Our approach can be used in order to detect arbitrage opportunities of any order. One may also be interested in detecting arbitrage opportunities of a particular order, that is, riskless profits which are the result of a strategy involving a given number of currencies.

Definition 5 *There is* S*-order arbitrage in the currency market if there exists* $\mathbf{x} \in \underline{R}$ *such that*

$$\mathbf{x}'\mathbf{l}^b \geq 0,$$

and

$$\mathbf{x}'\mathbf{1}_{N^2} = S, \tag{9}$$

where $\mathbf{1}_{N^2}$ *is a* N^2-*dimensional vector of ones.*

Let $\underline{R}^S \subset \underline{R}$ denote the set of recombining sequences involving S currencies and define

$$\theta_S \equiv \max_{\mathbf{x}} \left\{ \mathbf{x}'\mathbf{l}^b : \mathbf{x} \in \underline{R}^S \right\}. \tag{10}$$

The following characterization is trivial.

Theorem 6 *There is* S*-order arbitrage in the currency market if and only if* $\theta_S > 0$.

It is tempting to conclude that this type of arbitrage can be detected by just adding the constraint (9) to the problem that defines $\bar{\theta}$. As it turns out, this would not be enough. The potential presence of subsplits cannot be easily avoided now. For example, two triangular arbitrages strategies with nonoverlapping currencies may give the false idea that 6-order arbitrage is present. The problem may be even bigger. Consider again the vector $\mathbf{x}$ with components $x_{12} = x_{21} = x_{34} = x_{45} = x_{53} = 1$ and zero otherwise, and suppose that this is the optimal vector of such postulated maximization. It is clearly possible to have

$$\mathbf{x}'\mathbf{l}^b = l_{12} + l_{21} + l_{34} + l_{45} + l_{53} > 0$$

with $l_{12} + l_{21} < 0$. This vector satisfies the constraint (9) with $S = 5$ but this again is only a case of triangular arbitrage. Furthermore, as long as this triangular arbitrage is large enough, it may compensate the negative value of any other nonoverlapping recombining sequence and the maximization may produce a positive value for any $S \geq 5$.

There is a way to avoid this problem. Introduce a vector of real nonnegative variables $\mathbf{v} = (v_1, \ldots, v_N) \in \Re_+^S$ and consider the following set of constraints

$$v_i - v_j + (S+1)\, x_{ij} \leq S,\ \forall i, j = 2, \ldots, \underline{N},\ i \neq j. \tag{11}$$

The above conditions together with (9) and (2) guarantee that only recombining sequences in $\underline{R}^S$ which contain currency 1 are feasible (note that i and$j$ above cannot be equal to 1). For example, with $S = 5$, our vector x above with nonzero components $x_{12} = x_{21} = x_{34} = x_{45} = x_{53} = 1$ does not satisfy (11). To see this note that the inequalities

$$\begin{aligned} v_3 - v_4 + 6 &\leq 5 \\ v_4 - v_5 + 6 &\leq 5 \\ v_5 - v_3 + 6 &\leq 5 \end{aligned} \tag{12}$$

if added up give the wrong inequality

$$18 \leq 15.$$

Hence, the addition of (11) would do the trick if we were only interested in detecting S-order arbitrage which contains currency 1. Although, this case is of practical interest (a US trader may only be interested in finding arbitrage which could be implemented by starting and finishing the trade with US dollars), it does not fully address the subsplit issue. A slight modification of the inequalities in (11) delivers a feasible set containing all recombining sequences with S elements, rather than only those that include currency 1. Indeed, consider an N-dimensional vector of binary variables $\mathbf{y} = (y_1, \ldots, y_N)$ and the constraints given by

$$v_i - v_j + (S+1)(x_{ij} - y_i) \leq S, \; \forall i, j = 1, \ldots, \underline{N}, \; i \neq j, \tag{13}$$

and

$$\mathbf{y}' 1_N = 1. \tag{14}$$

Since $\mathbf{y}$ is a vector of binary variables, this last constraint imposes that only one element of y will be equal to 1 whereas the remaining ones will all be zero.

Let us see that this new set of constraints does in fact the trick. Suppose that $S = 5$ and consider a vector $\mathbf{x}$ with nonzero components $x_{23} = x_{34} = x_{45} = x_{56} = x_{62} = 1$. By choosing $y_2 = 1$, the constraints above associated with the nonzero elements of $\mathbf{x}$

are

$$
\begin{aligned}
v_2 - v_3 &\leq 5 \\
v_3 - v_4 + 6 &\leq 5 \\
v_4 - v_5 + 6 &\leq 5 \\
v_5 - v_6 + 6 &\leq 5 \\
v_6 - v_2 + 6 &\leq 5
\end{aligned}
$$

Each one of these constraints holds by choosing $v_2 = 6$, $v_3 = 2, v_4 = 3, v_5 = 4$, $v_6 = 5$. Their sum gives the correct inequality $24 \leq 25$. The other constraints in (13) hold as long as we set the remaining components of $\mathbf{v}$ equal to zero. By following similar arguments, any recombining $\mathbf{x}$ with 5 nonzero components can be proved to be feasible.

On the other hand, the vector x with nonzero components $x_{12} = x_{21} = x_{34} = x_{45} = x_{53} = 1$ will not satisfy the constraints. To see this, note that any vector $\mathbf{y}$ with $y_3 = y_4 = y_5 = 0$ will result in the set of inequalities in (12) which do not hold. Also, choosing for example $y_3 = 1$ gives

$$
\begin{aligned}
v_1 - v_2 + 6 &\leq 5 \\
v_2 - v_1 + 6 &\leq 5,
\end{aligned}
$$

a set of inequalities that does not hold either.

Since all the above arguments are easily generalized to any positive integer value of $S \leq N$. We have just proved the following result. Let B^Ndenote the set of $N-$dimensional vectors of bynary variables.

Theorem 7 *Let B^Ndenote the set of $N-$dimensional vectors of bynary variables. We have that*

$$\begin{aligned} \theta_S &= \max_{\mathbf{x}\in\underline{X},\mathbf{y}\in B^N,v\in\Re^N} \mathbf{x}'\mathbf{l}^b \qquad (15) \\ & \quad \textit{subject to } (7),\ \textit{(13) and}(14) \end{aligned}$$

The above theorem allow us to compute the maximum S-order arbitrage profit that can be obtained per unit traded and the optimal S-order arbitrage strategy. As we argued earlier, in most practical purposes and in the absence of negative spreads, Program 8 will deliver the optimal arbitrage strategy in $\underline{R}$. First, two nonoverlapping triangular arbitrage oportunites rarely coexist in real-life, let alone nonoverlaping arbitrage strategies of a higher order. Second, one can always add a constraint like

$$\mathbf{x}'\mathbf{1}_{N^2} \leq S \qquad (16)$$

to the feasible set of Program 8 with, for example, $S = 5$. In the absence of negative spreads, no subsplits will thus be possible in the optimal solution of $\mathbf{x}$ if there is arbitrage. In this case, the program delivers the optimal arbitrage strategy of an order less or equal than five. This approach will always work unless the optimal solution is a 6-order arbitrage strategy or of a higher order. If negative spreads are possible, subsplits may occur in the optimal solution of Program 8 whenever the number of nonzero elements in its optimal vector $\mathbf{x}$ is greater or equal than four.

Obviously, one can always follow a brute force approach by noting that

$$\theta = \max \{\theta_2, \ldots, \theta_N\}$$

and by solving Program 15 for all $S = 2, \ldots, N$. This may be an option the solution of Program 8 delivers subsplits.

Nevertheless, documenting the presence of arbitrage of a given order may be done by solving Program 15.

4.2 Arbitrage involving a given currency or quote

Our approach can be also used to investigate the quality of the price-setting process associated with a given currency or quote. Indeed, one can always add an additional constraints of the type

$$x_{ij} = 1$$

if the quality of the quote l_{ij}^b is to be examined or a constraint like

$$\sum_{i=1, i \neq j}^{N} x_{ij} = 1,$$

if the focus is on currency$j$. If one proceeds by adding any constraint of this type to the feasible set of Program 8, one must proceed with cautiousness because this constraints may force subsplits which may falsely attribute the presence of a given currency/quote in arbitrage strategies along the lines discussed in the previous section. The ways aroun follow identical arguments and one can rely on the extra constraint (16) or on a brute force approach.

5 Data

There are two interdealer trading venues offered by Electronic Broking Services (EBS) and Reuters. We use data from the EBS electronic trading platform. EBS was created by a partnership of the world's largest foreign exchange (FX) market making banks in 1990 to challenge Reuters' threatened monopoly in interbank spot foreign exchange and provide effective competition. EBS was acquired by ICAP[7] in 2006 and is the biggest interdealer trading venue for major currencies since mid-90s. Its position is particularly strong in major currencies. Table 1 shows the currencies being involved in most currency trading and the most traded currency pairs. Of the six most traded currencies, EBS has the dominant position in all of them except GBP.[8] Reuters

[7]ICAP plc is a UK-based voice and electronic dealer broker and provider of post trade risk services, the largest in the world carrying out transactions for financial institutions rather than private individuals.

[8]EBS had also a dominant position in major currencies and currency pairs before the introduction of the euro in 1999.

Dealing 3000 is the primary trading venue for commonwealth currencies[9] and for less traded currency pairs. All the major currency pairs are traded on EBS. Except CAD/USD, all these currency pairs can be involved in arbitrage strategies studied in this paper. In addition to the mentioned currency pairs, NZD/AUD and NZD/USD are also involved in arbitrage opportunities detected in our data. Using the EBS data we also see some other less traded currencies being involved in currency arbitrages.

Table 1: Most traded currencies and currency pairs (BIS, 2013)

EBS is organized as an electronic limit order book. Traders can submit limit orders (bid and ask). Alternatively, traders can "hit" bid or ask using a market order. Minimum trade size is one million of base currency. As part of the dealing rules, EBS facilitates each institution to manage its bilateral credit lines with counterparties. Each EBS-linked institution sets credit lines, including zero, against all other potential counterparties in the system. This means that an institution faces a restriction of tradable bids, asks or deals with other institutions. This also means that not all bids or offers are available for all institutions, as mentioned in the section about related literature. EBS requires that a new institution that is linked to EBS secures a sufficient number of other banks that are willing to open credit lines with the new

[9] In addition to exchange rates involving GBP, Commonwealth currencies also include trading in AUD/USD, NZD/USD and USD/CAD.

institution. Combined with the introduction of the clearing system (CLS) in 2002, settlement risk (credit risk) is very low in the interdealer market. This may also be an important reason for why the foreign exchange spot market worked very well during Fall 2008. Following the collapse of Lehman Brothers in September 2008, market sentiment deteriorated rapidly and money and capital markets were brought to a virtual standstill, however, the FX market continued to function without disruption throughout the market turmoil that followed.

Our EBS data set contains best bid and ask prices, depth at best bid and ask, and deals from January 1997 until August 2007. All bids and asks are tradable, and the best bid and ask is valid up to the specified depth. EBS also provides some information about the limit order book for levels outside the best bid and ask quotes. The settlement of foreign spot exchange transactions is set to be two business days later.

There have been some important structural changes with regard to trading technology. We will mention the most important changes. EBS was the first organization to facilitate algorithmic trading in spot FX. The "Ai trader" system was introduced in 2003 allowing banks to plug their computers with algorithmic trading programs directly to the EBS trading system computer. Before "Ai trader" was introduced, only humans could trade on the EBS platform. This innovation meant that execution can be done at much higher speed. Algorithmic trading can be used for e.g. taking advantage of arbitrage opportunities. The market share of "Ai trader" on EBS was close to zero during the first months in 2003, and was less than 5% during 2004. During 2005 the market share increased from 5% to 10% before the market share increased rapidly during 2006 and 2007. In May 2006 it was more than 20%, and in 2007 it became 50-60%. Since 2007 the market share of "Ai-trader" has been quite stable.[10] Chaboud et al. (2013) also present results suggesting that algorithmic models played an important role in three FX markets since 2005 (USD/EUR, JPY/USD and JPY/EUR).

The "primary customer" (PC) system was introduced in May 2004. The PC system allowed smaller banks with manual trading to make a contract to receive the service of large banks as primary brokers ("EBS Prime") The PC banks can make orders under the name of primary brokers to the EBS system. The primary broker collects fees, while primary customers can have narrower bid-ask spread since they obtain quotes from a larger set of counterparties. With this system, the small bank is benefitting from the use of the credit lines of the primary broker. The primary broker earns money from collecting the fee. Since the introduction of CLS and PC we observe only very few evidence of negative spreads (Ito, 2012).

In December 2004 the "Professional Trading Community" (PTC) was introduced.

[10] The information of market shares from 2003 until June 2006 is from ICAP. The information about market share in 2007 is from wikipedia, and is consistent with information obtained from conversations with people working in FX markets.

Until this date, only banks were allowed to trade at EBS. The introduction of PTC allowed players such as hedge funds, commodity trading advisors, proprietary trading houses, and other non-bank financial institutions to trade on EBS. Most of these players rely heavily on algorithmic trading. From December 2004 these players were allowed to connect their computers directly to the EBS trading platform. We expect computers to detect and exploit arbitrage opportunities faster than humans. We thus expect that the introduction of "Ai trader" and "PTC" would shorten duration of arbitrage opportunities. In June 2006, EBS Prime had 159 users of which 112 were bank customers and 37 non-bank customers.

The cost of running an EBS terminal involves a fixed set-up cost, monthly charges and trade fees. For EBS Prime Banks, Marshall et al. (2008) suggest that trade fee up to USD 1 billion is USD 7.50 per million, USD 5.00 per million for the next USD 9 billion, and USD 3.50 per million after that. Ito et al. (2012) report a trading fee of USD 2 per million for large volume banks, while small volume banks pay a fee of USD 7.50 per million. The most active institutions will thus have much lower fees than less active institutions. The EBS Prime Bank may charge a fee to EBS Prime Customers. There are no settlement fees with EBS.

When we examine arbitrage opportunities we do not include the transaction fees. First, these fees are small, for instance a trade fee of USD 5 million would e.g. reduce profit by 0.15 basis points ($3 \times 5/1000000$) for a triangular arbitrage. Second, major banks may have significantly lower fees since size of fees depends on their trading volume on EBS (e.g. 0.06 basis points with a fee of USD 2 per million). Notice also that EBS Prime banks earn money by charging trades fees from EBS Prime Customers. The trading by Prime customers increases the Prime bank's overall trading volume, and thus lower their trade fees paid to EBS. In addition, the EBS Prime Bank may obtain information regarding flow from observing trading by their EBS Prime Customers (e.g. Evans and Lyons, 2002).

We delete negative and zero spreads, although these opportunities may have been available for major banks with many credit lines. If all Prime banks had credit lines with each other, negative or zero spreads would not be possible. For instance, if a dealer submits a bid (limit order) that is equal or higher than the current ask, this would automatically generate a trade at the ask. Including observations with negative or zero spreads would obviously increase the number and average size average opportunities. Due to changes in market design described above, the number of zero and negative spreads have decreased significantly over time.

6 Results

Table 2 shows arbitrage strategies where we force the sale (Panel A) or purchase (Panel B) of a particular currency for the period 1999-2007.[11] For instance, in Panel A "USD/EUR" forces a sale of USD in the arbitrage strategy. A sale of USD will be executed at the bid. Similarly, in Panel B "USD/EUR" forces a purchase of USD which would be executed at the ask. The arbitrage opportunities are exploited using market orders, that is, the half-spread is paid for every transaction.

The second column in Table 2 shows the number of observations in which an arbitrage was feasible but not present. The third column shows the available number of exchange rates when there was an arbitrage opportunity. We see that there were typically seven available exchange rates traded on EBS when there was an arbitrage opportunity. The fourth column shows the number of detected arbitrage opportunities. The ratio of arbitrage opportunities as a fraction of feasible opportunities vary from 0.02% to 0.82%. We see that the ratio is highest for the most liquid currency pair (USD/EUR), and in general this ratio is higher for the most liquid currency pairs traded at EBS. We notice that the ratio is lower for the exchange rates involving GBP. One reason for this may be that the most active trading in GBP takes place at the Reuters Dealing 3000 platform.

Mean profits vary between 0.73 and 5.36 basis points, while the medians are

[11] Trading in euro began at the beginning of 1999. Before the euro, there were trading in e.g. german mark (DEM). We have thus skipped this period in this table.

around 0.60 basis points. This means that there is positive skewness. We also note that both averages and medians are significantly higher than trading fees.

Duration of an arbitrage opportunity is measured as the time that the quotes in the arbitrage strategy do not change. This is a conservative measure. For instance, if one quote in the arbitrage strategy changes, but do not eliminate the arbitrage, we will measure duration as the time between the arbitrage was detected until the change in quote. Average duration vary between 1.27 and 4.42 seconds. Arbitrage opportunities involving bid or ask of CHF/JPY typically have longer duration, on average, than arbitrage opportunities involving other currency pairs.

"Average number of currencies in arbitrage" shows the average number of currencies involved in the arbitrage. In a triangular arbitrage three currencies would be involved. Since the average number of currencies involved in an arbitrage is bigger than 3, this means that there are arbitrage opportunities involving more than three currencies. For USD/EUR (both bid and ask) the average is only slightly higher than 3. Other currency pairs are more often part of arbitrage opportunities with more than three currencies involved.

Table 2: Arbitrages involving bid (Panel A) and ask (Panel B) for a particular currency

Table 3 documents the development in arbitrage opportunities over time from 1997 until 2007. This table presents results for the best arbitrage strategies at every point in time, and thus avoid reporting the same arbitrage opportunity several times.[12] The first two years (1997-1998) represent the period before the introduction of the euro. The euro was introduced to world financial markets as an accounting currency on 1 January 1999, replacing the former European Currency Unit (ECU) at a ratio of 1:1 (USD 1.1743). Euro coins and banknotes entered circulation on 1 January 2002.

Table 3 shows that trading activity has increased significantly, while the number of arbitrage opportunities has not changed much over time. This means that the ratio arbitrage/feasible has decreased. In 1998 and 1999 the ratio was roughly 6%, decreasing to 3.5% in 2001, and to 0.8% in 2007. This trend may be related with some major changes in trading environments. First, in 2002 a new clearing system was introduced that reduced settlement risk (credit risk) to a minimum. In 2003, computerized trading was introduced and become popular over the next years. The

[12] To avoid reporting the same arbitrage opportunity several times we do the following: When an arbitrage opportunity is detected, we check the following observations and we stop when the opportunity disappears, that is, when there is any modification of the prices which are part of the arbitrage. Any observation in between the first time the arbitrage is spotted and the last time it persists will be disregarded. The only problem with this is that there may be other arbitrage opportunities in those in-between observations which involve other exchange rates different than the ones considered in the arbitrage detected in first place. However, this will seldom be the case and we can stick to this (conservative) way of reporting arbitrage.

"primary customer" (PC) system was introduced in 2004 allowing smaller banks to trade through large banks functioning as primary brokers ("EBS Prime"). The PTC system was also introduced in 2004 allowing players such as hedge funds, commodity trading advisors, proprietary trading houses, and other non-bank financial institutions to trade on EBS. Most of these participants rely on computerized trading. We notice that the ratio of arbitrage/feasible decreases from 3.5% in 2001 to 1.2% in 2005.

A possible consequence of algorithmic trading is that the size of average and median arbitrage has been reduced. Computers can quickly take advantage of arbitrage opportunities. Surprisingly, median and average durations of arbitrage opportunities have not change much since the end of the 1990s. However, median profits are low during the last years and may explain why duration does not change much. We also notice that the average number of currencies involved in an arbitrage is stable over time. However, we notice that the "number of available currencies when arbitrage" increases in 2007. During this year there were trading in more currency pairs on the EBS platform than during the previous years.

Table 3: Development in arbitrage opportunities 1997 - 2007

Our results in Table 3 are consistent with results from triangular arbitrages presented in other papers. Fenn et al. (2008) use data from HSBC for three time periods, one week in October 2003 and 2004, and 26 days in October 2005. They study triangular arbitrages involving EUR, USD, JPY and CHF. Both arbitrage profits and durations are similar to the numbers presented in Table 3. Marshall et al. (2008) use data from EBS for 2005 to study triangular arbitrage opportunities in EUR, USD CHF, GBP and JPY, and find evidence of similar arbitrage profits as in this study. Kozhan and Tham (2012) use data from Reuters trading system Dealing 3000 for three currency pairs (USD/EUR, USD/GBP and GBP/EUR) for 2003 and 2004.The size of average arbitrages is slightly higher, while duration is slightly lower.

Table 4 shows that the number of arbitrage opportunities vary with trading activity. The most active trading hours are from GMT 07:00 until 18:00. GMT 07:00 until GMT 12:00 represents a time period when the European market is open, while GMT 12:00 until GMT 16:00 represents a period when both the European and US market are open. GMT 16:00 until GMT 20:00 represents a period when the US market is open, but not the European market. The period from GMT 20:00 until GMT 24:00 is regarded as a quiet period. The Japanese market is open from GMT 00:00 until GMT 07:00. We see from Table 4 that the number of arbitrage opportunities is positively related with trading activity. Most arbitrage opportunities arise when both the European and the US markets are both open. Median profits and median durations are calculated from hourly averages. We see that the size of arbitrage opportunities seem to be slightly smaller during the more active trading hours. Duration of arbitrage opportunities are usually shorter during the more active periods.

Table 4: Seasonality in arbitrage opportunities

Table 5 studies the duration of arbitrage opportunities in detail. The table shows percentage of all arbitrage opportunities lasting for less than a second, 1-2 seconds, 2-3 seconds, 3-4 seconds, 4-5 seconds, and more than 5 seconds. We we would expect that the changes that took place between 2002 and 2004 would reduce not only size, but also duration of arbitrage opportunities. However, Tables 2 - 4 suggest that durations have not changed much. Surprisingly, more arbitrage opportunities last for one second or more in the last two periods (2002 - 2004 and 2005 - 2007) than in the previous periods. A possible explanation may be that arbitrage opportunities during the last two periods are, on average, small. Hence, e.g. execution risk may explain why duration of arbitrage opportunities has increased. Computerized trading can be executed within milliseconds. Latency may vary depending on location etc. However, all arbitrage opportunities that last for a second or more should be possible to exploit. For arbitrage opportunities that last for less than a second, not all arbitrage opportunities may be exploitable.

Table 5: Duration of arbitrage opportunities

Tables 6 and 7 analyze size of arbitrage opportunities in different time zones. As documented above, the average and median size of arbitrage opportunities have been reduced over time. In Table 6 we only focus on individual arbitrage opportunities for the last period from 2005 until 2007. The results document that during this period, most of the arbitrage opportunities are below 0.5 basis points. However, there are also many arbitrage opportunities above 0.5 and 1 basis points. Mean and median durations are sufficiently long such that many of these arbitrage opportunities would be possible to exploit. Notice that durations are shorter during more active trading hours for arbitrage opportunities for arbitrage opportunities in all size categories. Arbitrage opportunities involving more than three currencies show much the same pattern as triangular arbitrage strategies.

Table 6: Size of triangular arbitrage opportunities in different time zones

Table 7: Size arbitrage opportunities involving more than three currencies in different time zones

6.1 Regressions

In this subsection we provide some evidence on how size, duration and frequency of arbitrage opportunities vary with the pace of the market and with market volatility. Furthermore, we control for important changes in market structure. First, we the introduction of PC that took place at he same time as algorithmic trading became possible (end of 2003/beginning of 2004). Second, PTC was introduced at the end of 2004. We also include a variable that capture the possibility for arbitrage opportunities including more than three currencies.

We estimate three regressions including size of arbitrage opportunities, duration and frequency of arbitrage opportunities as dependent variables, regressed on an intercept, a simple proxy for liquidity, a proxy for market volatility, a variable capturing the possibility for arbitrage through more than three currencies, and two dummy variables (one for the introduction of PC and algorithmic trading and one for the introduction of PTC). The models are estimated with GMM.

Table 8 presents our result. Our results suggest that the size of arbitrage opportunities increases with the size of the bid and ask spread. The bid and ask spread is typically low when trading is active. This result is also consistent with prediction that execution risk increases when liquidity is low (Kozhan and Tham, 2012). The size of arbitrage opportunities do also increase with market volatility. This finding may e.g. be consistent with physical dealers that are not updating their limit orders fast enough in volatile markets.

The size of arbitrage opportunities increases when arbitrage through more currencies become possible. The dummy variables have the expected signs. Both the introduction of algorithmic trading (and PC) and PTC have negative coefficients.

Table 8: Estimated relationships between the characteristics of arbitrage opportunities and liquidity and market volatility

Duration of arbitrage opportunities increases with the bid and ask spread, and decreases with market volatility. Furthermore, the ratio of the number of arbitrage opportunities over feasible arbitrages increases with market volatility and with the number of currencies that may enter into arbitrage strategy. The ratio has, as expected, decreased significantly since the introduction of algorithmic trading.

6.2 Considerations regarding trading costs etc.

We have not included trade fees in our analysis. Trade fees are different for different institutions. Large Prime Banks have very low fees. For instance, a trade fee of USD 2 would reduce profit by 0.06 basis points in a triangular arbitrage, while less active banks may pay a fee of e.g. USD 7.50 per trade (that is, 0.225 basis points for a triangular arbitrage). PC banks and PTC institutions have access via a prime bank and will pay an additional fee to the bank acting as primary broker (EBS Prime).

Latency costs may be important. There is no guaranty that the arbitrage transaction will be completed such that there will be execution risk. In equilibrium there is a positive relation between execution risk and market illiquidity (Kozhan and Tham, 2012). Our regression results support that execution risk may be important. These costs may also vary among prime banks, PC banks and PTC institutions. Large banks with faster infrastructure (and better location with regard to the hub) will have faster access and thus lower latency costs. Also, arbitrages involving more currencies have more execution risk than triangular arbitrage strategies.

Arbitrageurs will have some residual position exposure since all trades can be carried out only in multiples of one million units of the base currency on EBS. The residual will always be smaller than a million units, the arbitrageur can clear the accumulated residual when it exceeds one million units. This means that the arbitrageur is exposed to price fluctuations when clearing the residual position. It is also possible to assume that the arbitrageur manage inventory at regular time intervals. An alternative would be to use more than one trading channel. There are many different trading venues. Both multibank and single bank platforms allow for trades below 1 million units, and will normally provide competitive quotes. Major banks do most of their inventory management internally, and may thus have much smaller inventory costs than small banks and PTC institutions.

We have used tradable quotes from EBS. Combining EBS data with data from other trading venues would obviously increase the number of arbitrage opportunities. For instance, combining EBS data with Reuters tradable quotes would mean improved quotes for exchange rates involving GBP. Recently, there has become active trading in many different venues, multiplatforms and single bank platforms (Autobahn, Currenex etc.). These platforms offer quite tight spreads, and do also offer trading opportunities in currency crosses not traded at the EBS or Reuters platform. By incorporating more sources of tradable quotes in the analysis, we would probably increase the size and the number of arbitrage opportunities, and also increase the number of arbitrage opportunities involving more than three currencies. In this paper, however, we want to evaluate the use of our model and not prove big arbitrage opportunities.

7 Conclusions

Finance theory suggest that in well-functioning markets no-arbitrage conditions should hold continuously. This paper provide evidence of short-lived arbitrage opportunities in major FX markets. The size of the arbitrage opportunities may be economically significant and is unlikely due to transaction costs, and durations are often high enough to exploit these arbitrages. Furthermore, we find that the average size of arbitrages has decreased over time, but there are still significant arbitrage opportunities toward the end of our dataset. Using tradable quotes from EBS over the over the period 1997-2007 we document several short-lived arbitrage involving up to five currencies.

In this paper we show that the existence of arbitrage of any order, involving any number of currencies, can be treated within a simple general framework. This general type of arbitrage can be detected by using a simple linear program, and the optimal strategy (the one providing the highest riskless profit by monetary unit traded) can be computed by using a simple mixed-linear program. We think our contribution may also be relevant for real-world application.

We see several possible extensions of this paper. In this paper, we only focus on arbitrage opportunities using market orders. An alternative strategy would e.g. use strategies also involving limit orders. For instance, a strategy would be to submit limit orders in such a way that when a limit order is hit, the arbitrage strategy is executed by two or more market orders. The advantage is that you would not pay half-spread for all trades in an arbitrage strategy, but instead earn the half-spread for at least one of the trades.

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Table 1: Most traded currencies and currency pairs (B.I.S, 2013)

	2001	2004	2007
Currencies:			
USD	89.9	88.0	85.6
EUR	37.9	37.4	37.0
JPY	23.5	20.8	17.2
GBP	13.0	16.5	14.9
CHF	6.0	6.0	6.8
AUD	4.3	6.0	6.6
Exchange rates:			
USD/EUR	30.0	28.0	26.8
JPY/USD	20.2	17.0	13.2
USD/GBP	10.4	13.4	11.6
AUD/USD	4.1	5.5	5.6
CHF/USD	4.8	4.3	4.5
CAD/USD	4.3	4.0	3.8
JPY/EUR	2.9	3.2	2.6
GBP/EUR	2.1	2.4	2.1
CHF/EUR	1.1	1.6	1.9
Total	79.9	79.4	72.1
Others	20.1	20.6	27.9

"Currencies" show the percentage of all trades that involve the particular currency. "Exchange rates" show the percentage of trades that take place in that particular currency pair.

Table 2: Arbitrages involving bid (Panel A) and ask (Panel B) for a particular currency

	All	Arbitrages							
		Avail. Exch. Rates		Arbitrages/	Profits		Duration		Av. no of curr.
	feasible	when arbitrage	Total	Feasible	Mean	Median	Mean	Median	in arbitrage
Panel A Arbitrage opportunities involving bid quotes:									
USD/EUR	256765429	6.79	2111580	0.82%	0.91	0.57	1.49	1.21	3.004
JPY/EUR	260021263	6.97	1355797	0.52%	0.80	0.59	1.38	1.03	3.231
CHF/EUR	262407518	6.81	1447138	0.55%	0.90	0.59	1.49	1.18	3.376
GBP/EUR	204206491	6.94	199555	0.10%	1.28	0.60	1.76	1.28	3.665
JPY/USD	259327931	6.84	1567201	0.60%	1.00	0.60	1.58	1.05	3.253
CHF/USD	263608011	6.81	1189982	0.45%	0.73	0.58	1.27	1.15	3.327
GBP/USD	204392140	7.47	205543	0.10%	1.31	0.57	1.95	1.22	3.725
CHF/JPY	9932131	9.42	2062	0.02%	1.58	0.66	3.98	1.80	3.338
Panel B Arbitrage opportunities involving ask quotes:									
USD/EUR	256625552	6.86	2068619	0.81%	0.76	0.58	1.47	1.23	3.005
JPY/EUR	259564945	6.82	1565122	0.60%	0.99	0.60	1.65	1.14	3.253
CHF/EUR	263655943	6.82	1192317	0.45%	0.75	0.57	1.27	1.12	3.329
GBP/EUR	202699202	7.44	137306	0.07%	0.86	0.56	1.44	1.31	3.547
JPY/USD	259822289	6.95	1350235	0.52%	0.78	0.58	1.38	1.01	3.230
CHF/USD	262344135	6.81	1447091	0.55%	0.92	0.58	1.49	1.15	3.375
GBP/USD	202902561	7.40	133476	0.07%	0.96	0.56	1.44	1.30	3.581
CHF/JPY	9738871	8.34	3191	0.03%	5.36	0.63	4.42	1.33	3.351

"All feasible" shows the number of observations in which the arbitrage was feasible but not present. "Avail. Exch. Rates when arbitrage" shows the average number of exchange rates present when there was an arbitrage opportunity. "Total" is the total number of arbitrage opportunities. "Arbitrage/Feasible" is the percentage of arbitrage opportunities as a fraction of feasible opportunities. "Profits Mean" represents the average size of an arbitrage opportunity, while "Profits Median" represents the median based on daily averages. "Duration Mean" and "Duration Median" represent the average duration in seconds and median duration

Table 3: Development in arbitrage opportunities 1997 - 2007

			Arbitrages						
	All	Avail. Curr.		Arbitrages/	Profits		Duration		Av. No of curr.
Year	feasible	when feasible	Total	Feasible	Mean	Median	Mean	Median	in arbitrage
1997	10344671	6.34	613274	5.93%	1.35	0.69	2.09	1.71	3.32
1998	11377346	5.81	741872	6.52%	1.19	0.73	1.56	1.39	3.31
1999	10017467	5.91	451504	4.51%	0.95	0.68	1.34	1.23	3.30
2000	13459507	6.34	497536	3.70%	1.08	0.69	1.49	1.30	3.36
2001	14105374	6.53	498758	3.54%	0.97	0.63	1.33	1.23	3.29
2002	14037927	6.57	415185	2.96%	0.73	0.52	1.43	1.22	3.27
2003	22251915	6.28	498385	2.24%	0.68	0.51	1.36	1.35	3.31
2004	28375405	6.74	492498	1.74%	0.65	0.48	1.19	1.27	3.37
2005	38775714	6.92	481195	1.24%	0.54	0.39	1.46	1.29	3.31
2006	45721555	6.96	532374	1.16%	0.76	0.30	1.84	1.20	3.25
2007	63386727	9.14	509607	0.80%	0.62	0.28	1.68	1.15	3.24

"All feasible" shows the number of observations in which the arbitrage was feasible but not present. "Avail. Exch. Rates when arbitrage" shows the average number of exchange rates present when there was an arbitrage opportunity. "Total" is the total number of arbitrage opportunities. "Arbitrage/Feasible" is the percentage of arbitrage opportunities as a fraction of feasible opportunities. "Profits Mean" represents the average size of an arbitrage opportunity, while "Profits Median" represents the median based on daily averages. "Duration Mean" and "Duration Median" represent the average duration in seconds and median duration in seconds based on daily averages, respectively. "Av. no. of curr. in arbitrage" is the average number of currencies represented in an arbitrage opportunity.

Table 4: Seasonality in arbitrage opportunities

	2005 - 2007 Arbitrages				2002 - 2004 Arbitrages				1997 - 2001 Arbitrages			
	Arbitrage/ feasible	Arb. per year	Median profit	Median Dur	Arbitrage/ feasible	Arb. per year	Median profit	Median Dur	Arbitrage/ feasible	Arb. per year	Median profit	Median Dur
00-01	1.01%	25959	0.44	1.71	1.65%	20106	0.57	1.62	2.35%	25506	0.75	1.69
02-03	0.79%	15637	0.32	1.40	1.42%	11055	0.44	1.50	3.34%	13033	0.60	1.50
04-05	0.78%	17498	0.31	1.33	1.47%	13358	0.46	1.47	3.41%	17439	0.64	1.31
06-07	1.00%	51371	0.30	0.98	2.02%	48860	0.48	1.29	5.14%	54227	0.65	1.31
08-09	1.06%	61666	0.29	0.98	2.22%	62418	0.46	1.30	5.76%	67065	0.65	1.47
10-11	0.98%	50729	0.29	1.16	2.02%	49871	0.47	1.30	5.36%	53055	0.68	1.59
12-13	0.98%	88876	0.29	1.16	2.02%	84622	0.47	1.30	5.36%	79453	0.68	1.59
14-15	1.21%	96145	0.31	0.82	2.71%	97199	0.52	0.92	5.75%	88841	0.69	1.10
16-17	0.88%	42129	0.31	1.20	2.03%	29728	0.56	1.20	3.96%	43140	0.72	1.31
18-19	0.86%	27006	0.34	1.50	1.84%	15521	0.57	1.44	2.70%	19667	0.72	1.50
20-21	1.04%	16022	0.42	2.33	1.78%	7813	0.60	2.17	2.13%	8668	0.72	2.31
22-23	1.01%	14687	0.44	1.71	1.65%	6045	0.57	1.62	2.35%	8111	0.75	1.69

"Arbitrage/Feasible" is the percentage of arbitrage opportunities as a fraction of feasible opportunities. "Arb. per year" is the average number of arbitrage opportunities per year. "Median profit" represents the median based on hourly averages. "Median Dur" represents the median duration in seconds based on hourly averages. "00-01" and "00-02" etc. represent the time interval based on GMT.

Table 5: Duration of arbitrage opportunities

	1997 - 1998		1999 - 2001		2002 - 2004		2005 - 2007	
Duration (sec.)	Percentage	Cumulative	Percentage	Cumulative	Percentage	Cumulative	Percentage	Cumulative
0	50.9%	50.9%	51.5%	51.5%	44.0%	44.0%	38.1%	38.1%
1	18.2%	69.0%	22.3%	73.8%	30.3%	74.3%	37.5%	75.6%
2	11.0%	80.0%	11.6%	85.4%	13.5%	87.7%	10.4%	86.0%
3	6.4%	86.5%	5.6%	90.9%	5.3%	93.0%	4.6%	90.6%
4	3.8%	90.2%	2.9%	93.8%	2.4%	95.4%	2.5%	93.1%
5	2.4%	92.6%	1.7%	95.5%	1.3%	96.7%	1.5%	94.6%
> 5	7.4%	100.0%	4.5%	100.0%	3.3%	100.0%	5.4%	100.0%

"Duration" is measured in seconds. "Percentage" is the percentage of all arbitrage opportunities with a particular duration. "Cumulative" just cumulates the probabilities.

Table 6: Size of triangular arbitrage opportunities in different time zones (2005 - 2007)

	Size of arbitrage	No of arbitrages	Duration Mean	Duration Median
Europe	07:00 - 12:00			
	<0.5	248211	1.34	1
	≥0.5<1	59790	1.43	1
	>1	49566	2.64	1
Europe and USA	12:00 - 16:00			
	<0.5	310392	1.03	1
	≥0.5<1	83761	1.02	1
	>1	71805	1.64	1
USA	16:00 - 20:00			
	<0.5	118158	1.78	1
	≥0.5<1	33641	1.80	1
	>1	27487	3.29	1
Japan and Australia	20:00 - 07:00			
	<0.5	182613	2.32	1
	≥0.5<1	55340	2.17	1
	>1	57581	3.54	1

"Size of arbitrage" separates profit opportunities into three groups: less than 0.5 basis point, from 0.5 to 1 basis point, and from 1 basis point. "No. of arbitrages" represent the number of arbitrages. "Duration Mean" and "Duration Median" represent the average and median duration based on individual arbitrage opportunities, respectively.

Table 7: Size arbitrage opportunities involving more than three currencies in different time zones (2005-2007)

	Size of arbitrage	No of arbitrages	Dur Mean	Median
Europe	07:00 - 12:00			
	<0.5	44734	1.37	1
	$\geq 0.5<1$	13345	1.22	1
	>1	12926	1.60	1
Europe and USA	12:00 - 16:00			
	<0.5	54979	1.04	1
	$\geq 0.5<1$	16715	0.89	1
	>1	17340	1.09	1
USA	16:00 - 20:00			
	<0.5	17048	1.87	1
	$\geq 0.5<1$	5625	1.45	1
	>1	5431	2.36	1
Japan and Australia	16:00 - 20:00			
	<0.5	15078	1.92	1
	$\geq 0.5<1$	5372	1.66	1
	>1	5402	2.11	1

"Size of arbitrage" separates profit opportunities into three groups: less than 0.5 basis point, from 0.5 to 1 basis point, and from 1 basis point. "No. of arbitrages" represent the number of arbitrages. "Duration Mean" and "Duration Median" represent the average and median duration based on individual arbitrage opportunities, respectively.

Table 8: Estimated relationships between the characteristics of arbitrage opportunities and liquidity and market volatility

	Size of Arbitrage		Duration		Arbitrage/ Feasible	
Bid_ask	0.159	**	0.389	***	0.185	
	2.530		3.046		0.155	
Stdev	0.449	**	-1.015	***	1.597	**
	7.183		-7.967		3.648	
No of curr.	0.032	**	-0.001		0.506	**
	2.347		-0.019		2.981	
D_PC	-0.194	**	0.104		-1.919	**
	-6.126		1.820		-3.666	
D_PTC	-0.083	**	0.059		0.415	
	-3.385		0.772		1.076	
Constant	0.032	**	1.006	**	2.770	
	2.347		6.039		1.601	
Adj. R^2	0.150		0.110		0.056	
Obs	2341		2341		2341	

The dependent variables are the size of arbitrage opportunities ("Size of Arbitrage") in basis points, duration ("Duration") of arbitrage opportunities measured in seconds and the ratio of number of arbitrage opportunities over the number of feasible arbitrages ("Arbitrage/Feasible"). "Bid_ask" is the (logarithm of) ask minus bid multiplied by 10000 for USD/EUR. "Stdev" is the standard deviation of USD/EUR multiplied by 100. "No. of curr." represents the (logarithmic) change in average number of currencies that may be involved in an arbitrage multiplied by 100. "D_PC" is a dummy variable taking the value one for the period after PC and algorithmic trading were introduced, while "D_PTC" is a dummy taking the value 1 after the introduction of PTC. The regressions are estimated using GMM using daily data.