ANALYSING COMMON STOCKS PERFORMANCE FROM THE OPTIMAL EX-POST PORTFOLIO WEIGHTS

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Maria-Teresa Bosch-Badia
Visiting Researcher
Universitat de Girona
Department of Economics
Campus de Montilivi
17071 Girona
email: teyabosch@gmail.com

Joan Montllor-Serrats
Professor of Finance
Universitat Autonoma de Barcelona
Department of Business
Campus de Bellaterra
08193 Bellaterra (Cerdanyola del Vallès), Barcelona
email: Joan.Montllor@uab.es

Maria-Antonia Tarrazon-Rodon (corresponding author)
Associate Professor of Finance
Universitat Autonoma de Barcelona
Department of Business
Campus de Bellaterra
08193 Bellaterra (Cerdanyola del Vallès), Barcelona
email: MariaAntonia.Tarrazon@uab.es

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ABSTRACT

How does the market regard the competitiveness of common stocks? In this paper we propose the optimal weight of each stock in the optimal portfolio calculated from the ex-post data as a new performance measure for common stocks. The optimal weight becomes a function of the part of the risk premium that excludes the asset contribution to the systematic risk of other assets (crossed beta). The ex-post optimal weight depends on alpha, crossed beta and the weight changes in the optimal ex-post portfolio with respect to the ex-ante portfolio. The analysis is developed for the mean-variance and Gini portfolio paradigms.

Keywords: Common stocks performance, beta, Gini portfolios.

JEL code: G11

1. Introduction

The financial performance analysis of common stocks has received substantially less attention than the performance analysis of investment funds. Nevertheless, the relevance of common stocks as financial assets justifies performance analysis specifically focused on them. Common stocks have some defining features that make their performance analysis different from investment funds. Hardly a single stock constitutes the main asset of a single investor in a long period of time. Instead, it is combined with other stocks inside a portfolio. Undiversified long positions in them generally obey to speculative strategies. The interest of common stocks performance analysis is not only for investors, but also for corporate managers. For a corporation, the comparison of the performance of its shares in the stock market with other shares of its industry and the stock market as a whole constitutes a highly valuable piece of information that enlightens how the market judges its long run strategic decisions and its short run management. Focusing on a corporation concerned with the competitiveness of its stocks as investment assets, there are two central questions to be asked: How does the market regard the competitiveness of these stocks with respect to the other stocks in the market? Can we find a quantitative measure for this competitiveness? The aim of this article is to answer these questions.

Among the classical performance measures, the Treynor ratio (Treynor, 1966) is the appropriate one to evaluate the return/risk relationship of an asset that is part of a portfolio hold by an investor (Bodie, Kane and Marcus 2009, p. 830; Amenc and Le Sourd 2003 p. 109). In these cases, Jensen's alpha (Jensen, 1969) also provides a valuable piece of information, but in order to make comparable the alphas of the assets that belong to a portfolio it is necessary to submit them to the beta equalizing procedure depicted by Bodie, Kane and Marcus (2009, pp. 829-830). Later developments on performance measurement have been centrally focused on investment funds and often on the incorporation of new risk measures to performance indicators. These are, among others, the cases of Sortino-Satchell ratio (Sortino and Satchell 2001), Omega ratio (Shadwick and Keating, 2002), Rachev ratio (Stoyanov, Rachev and Fabozzi, 2007), Farinelli and Tiblietti (2008) and Homm-Pigorsch ratio (2012). Bacon (2013) presents a panoramic development of performance measures.

Centring on stocks, there is a property that, at least according to our findings, has not been explored as a performance measure: The optimal weight of the stock under analysis in the ex-post optimal portfolio. The ex-ante weights, obtained from the expected data, inform investors about the percentage of their budget to be put on each stock. What we propose is this article is to recalculate the optimal weights on the basis of ex-post data and to take them as performance indicators. As we show in the text, the ex-post weights embed Jensen's alpha calculated, as usual, from the ex-ante weights and the ex-post data. For each stock, the ex-post optimal portfolio weight is the answer to the following question: Which would have been the optimal weight in the stock if the ex-ante data had been equal to the ex-post data? This question can be regarded as the concretion of the two previous ones that we have formulated: The more competitive an asset is, the larger should be its share in investors' portfolios. The optimal weight tells us how large it should be. The analysis of the elements that justify the optimal weight is central when we aim to take this proportion as a performance measure. To this end, we present a detailed formula for the optimal weights in which we introduce two new parameters: the crossed volatility and the crossed beta. They express, respectively, the parts of the covariance and beta that are due to the interaction of the stock under analysis with the other stocks in the market, excluding the proportion of this stock in the optimal portfolio. These parameters, together with the risk premia and the volatilities of the asset and the market explain, through an analytical relationship, the optimal weight of each stock in the optimal portfolio.

We adopt a twofold methodological approach. First, we focus on the mean-variance paradigm by studying the optimal weight of each asset in the CAPM market portfolio from the perspective of a performance measure. Next, we extend our analysis to the mean-Gini approach to portfolio selection studying again the optimal weights from the performance perspective. As pointed out by Yitzhaki (1982) and Shalit and Yitzhaki (1984, 2005, 2010), the mean-Gini approach has the advantage, with respect to the mean-variance approach, of being compatible with the second order conditions of stochastic dominance. Furthermore, the Gini approach also admits to incorporate investors' risk preferences (Yitzhaki and Schechtman, 2013, chp.6). Many performance indicators can be taken as the objective function of portfolio optimization models, as it has been shown by Farinelli et alii (2008, 2009). Then, in the approach we adopt in the present article, the optimal weights of the assets inside the portfolio can be regarded as second order performance indicators, i.e. the performance of the assets inside the performance of the portfolio. In this respect, it is worth taking into account the equivalences and divergences among performance measures, as studied by Eling and Schuhmacher (2007).

The article is organized as follows. In Section 2 we analyse the meanings of the optimal weights in the mean-variance approach. Section 3 is focused on the study of the ex-post optimal weights as a performance measure continuing with the mean-variance approach. Section 4 switches the analysis to the mean-Gini model. Section 5 presents an empirical illustration for the DJIA components in 2013 applying the mean-variance and the portfolio Gini analysis. Section 6 concludes the article.

2. Analyzing the meanings of the optimal weights in the mean-variance approach

The aim of this section is to analyze and interpret the meaning of the optimal weights in the mean-variance paradigm. The CAPM leads to an optimal portfolio, the market portfolio, that stems from obtaining the best combinations of risky stocks and the riskless asset. As known, the market portfolio includes all stocks in the market in the proportions of their values. These proportions are obtained by combining their expected rates of return, standard deviations and correlations coefficients in the framework of the Markowitz model. The optimal combinations create the efficient frontier of risky securities. The tangency between this frontier and the capital market line, in turn, creates the market portfolio. As known, the term *market portfolio*, in the CAPM context, refers to the optimal combination that includes all stocks in the market. Often, the market index is taken as a proxy for this term, although it has some theoretical shortcomings. The optimal portfolio that we obtain in this article can be regarded as an ex-post optimal market portfolio, because the optimization we develop is mainly thought to be applied to market indices, although it can also be applied to other portfolios. For these reasons we use indistinctively the terms *market portfolio* and *optimal portfolio*. Often, the Markowitz model includes in its optimization a restriction that discards short sales. Nevertheless, if our interest now is to analyze the meaning of optimal weights, it is essential not to include any restriction that forbids short sales, because we have to interpret as well the negative positions.

The optimization problem can be set out as minimizing the variance of the market portfolio subject to a specific rate of return and to the budget restriction, i.e. equating to 1 the sum of the proportions invested in the stocks and, usually, to the restriction that forbids short sales, which we exclude in this work. After the introduction of the risk free asset, the mean variance problem consists of the maximization of the Sharpe ratio of the portfolio subject to the budget constraint. It can be solved through Langrangian multipliers[1], but also (Elton and Gruber 1995, pp. 98-99) by maximizing the Sharpe ratio of the portfolio $\left(S_M\right)$ written as:

$$S_M = \frac{\sum_{j=1}^{n} x_j \left(\bar{R}_j - r\right)}{\sigma_M} \qquad (1)$$

where x_j denotes the weight of asset j in the portfolio, $\bar{R}_j$ its expected rate of return, σ_M the standard deviation the portfolio and r the risk free interest rate. Since we aim to determine the weight of a generic asset i in the portfolio, we write (1) by making explicit the weight of this asset $\left(x_i\right)$ in the numerator and in the denominator:

$$S_M = \frac{x_i\left(\bar{R}_i - r\right) + \sum_{j=1 \forall j \neq i}^{n} x_j\left(\bar{R}_j - r\right)}{\left(x_i^2\sigma_i^2 + \sum_{j=1 \forall j \neq i}^{n} x_j^2\sigma_j^2 + \sum_{i=1}^{n}\sum_{j=1 \forall j \neq i}^{n} x_i x_j Cov\left(R_i, R_j\right) + \sum_{\forall \left(\substack{j, j' \neq i \\ j \neq j'}\right)}\sum x_j x_{j'} Cov\left(R_j, R_{j'}\right)\right)^{1/2}} \qquad (2)$$

$Cov\left(R_i, R_j\right)$ denotes the covariance between R_i and R_j; $Cov\left(R_j, R_{j'}\right)$, in turn, denotes the covariance between and R_j and $R_{j'}$.

Deriving S_M with respect to x_i, equating the result to zero and solving for x_i (see appendix), we arrive at:

$$x_i = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)} \frac{\sigma_M^2}{\sigma_i^2} - \frac{1}{\sigma_i^2} \sum_{j=1 \forall j \neq i}^{n} x_j Cov\left(R_i, R_j\right) \qquad (3)$$

The same result can be obtained after performing some operations on the Security Market Line (see appendix).

Let us focus on the term $\sum_{j=1 \forall j \neq i}^{n} x_j Cov\left(R_i, R_j\right)$. The covariance between each stock and the optimal portfolio consists of the sum of the direct contribution of the stock expressed by the product of its weight by its variance ($x_i \sigma_i^2$), and the crossed effect expressed by the addition of the products of the weights of the rest of the stocks by their covariances with the stock under consideration. We call the latter *crossed volatility* $\left(CV\right)$.Thus:

$$Cov\left(R_i, R_M\right) = x_i \sigma_i^2 + \sum_{j=1 \forall j \neq i}^{n} x_j Cov\left(R_i, R_j\right) \qquad (4)$$

and:

$$CV_i = \sum_{j=1 \forall j \neq i}^{n} x_j Cov\left(R_i, R_j\right) \qquad (5)$$

With this equation in mind, we realize that beta can be written as the addition of two components:

$$\beta_i = \frac{x_i \sigma_i^2}{\sigma_M^2} + \frac{\sum_{j=1 \forall j \neq i}^{n} x_j Cov\left(R_i, R_j\right)}{\sigma_M^2} \qquad (6)$$

Which, respectively express the part of beta due to the volatility of the asset and the part of beta due to the covariances between the asset and the other assets in the market portfolio. We call the former *self-generated beta* $\left(\beta_{i\,i}\right)$,and the latter *crossed beta* $\left(\beta_{i\,j}\right)$:

$$\beta_{i\,i} = \frac{x_i \sigma_i^2}{\sigma_M^2} \tag{7}$$

$$\beta_{i\,j} = \frac{\sum_{j=1 \forall j \neq i}^{n} x_j Cov\left(R_i, R_j\right)}{\sigma_M^2} \tag{8}$$

$$\beta_i = \beta_{i\,i} + \beta_{i\,j} \tag{9}$$

On this basis, and after multiplying and dividing $Cov\left(R_i, R_j\right)$ by σ_M^2 in (3) and rearranging the equation, we can write the optimal weight as:

$$x_i = \frac{\left(\bar{R}_i - r \middle/ \bar{R}_M - r\right) - \beta_{i\,j}}{\left(\sigma_i^2 \middle/ \sigma_M^2\right)} \tag{10}$$

Performing some simple algebraic operations, equation (10) can be presented as:

$$x_i = \frac{\sigma_M^2}{\sigma_i^2} \frac{\left(\bar{R}_i - r\right) - \left(\bar{R}_M - r\right)\beta_{i\,j}}{\bar{R}_M - r} \tag{11}$$

Taking into account that beta can be expressed as the addition of self-generated and crossed betas, the expected risk premium[2] can also be broken down into its self-generated and crossed parts:

$$\left(\bar{R}_i - r\right) = \left(\bar{R}_M - r\right)\beta_i = \left(\bar{R}_M - r\right)\beta_{i\,i} + \left(\bar{R}_M - r\right)\beta_{i\,j} \tag{12}$$

Then, the expected self-generated risk premium $\left(\bar{\pi}_i\right)$ can be written as:

$$\bar{\pi}_i = \left(\bar{R}_M - r\right)\beta_{i\,i} = \left(\bar{R}_i - r\right) - \left(\bar{R}_M - r\right)\beta_{i\,j} \tag{13}$$

Substituting (13) in (11), we obtain that the optimal weight is a function of the self-generated risk premium:

$$x_i = \frac{\sigma_M^2}{\sigma_i^2} \frac{\bar{\pi}_i}{\bar{R}_M - r} \qquad (14)$$

The self-generated risk premium can, in turn, be adjusted to variances by multiplying it by the ratio $\left(\sigma_M^2 / \sigma_i^2\right)$ obtaining the variance adjusted self-generated risk premium ($\hat{\pi}_i$):

$$\hat{\pi}_i = \frac{\sigma_M^2}{\sigma_i^2} \bar{\pi}_i \qquad (15)$$

This result, enables us to express the proportion of asset i in the optimal portfolio as the proportion of the variance adjusted self-generated risk premium to the market risk premium:

$$x_i = \frac{\hat{\pi}_i}{\bar{R}_M - r} \qquad (16)$$

Recalling that the sum of weights equates 1, we can also say that the market risk premium is equal to the addition of the variance adjusted risk premia of its assets[3]:

$$\sum_{j=1}^{n} x_j \left(\bar{R}_M - r\right) = \left(\bar{R}_M - r\right) = \sum_{j=1}^{n} \hat{\pi}_j \qquad (17)$$

It stems from (11) that the condition for an asset being in long position consists of a total asset risk premium, $\left(\bar{R}_i - r\right)$, greater than its crossed risk premium:

$$\left(\bar{R}_i - r\right) > \left(\bar{R}_M - r\right)\beta_{i\,j} \qquad (18)$$

which, as it stems from (13), is equivalent to $\beta_{i\,i} > 0$ for positive market risk premia. Therefore, for positive market portfolio risk premia, short positions happen when the average risk premium is lower than the part of the required risk premium due to crossed beta: $\left(\bar{R}_i - r\right) < \left(\bar{R}_M - r\right)\beta_{i\,j}$. This condition is softer than a Treynor ratio greater than the market one. In effect, a stock has a Treynor ratio greater than the market one if its risk premium is higher than the market risk premium multiplied by beta

$\left(\left(\bar{R}_i - r\right) > \left(\bar{R}_M - r\right)\beta_i\right)$; but in our case we simply have to multiply the market risk premium by crossed beta.

Equation (14) also shows that the optimal weight depends on the specific risk. Let us recall that total risk $\left(\sigma^2\right)$ consists of the addition of systematic risk $\left(\beta_i^2\sigma_M^2\right)$ and specific risk $\left(\sigma_{\varepsilon i}^2\right)$. Then, the variance ratio in (14), can be written as:

$$\frac{\sigma_M^2}{\sigma_i^2} = \frac{\sigma_M^2}{\beta_i^2\sigma_M^2 + \sigma_{\varepsilon\, i}^2} = \frac{1}{\beta_i^2 + \frac{\sigma_{\varepsilon\, i}^2}{\sigma_M^2}} \tag{19}$$

Comparing (14) with (19) we draw the conclusion that the optimal weight is a decreasing function of specific risk. This property can be interpreted saying that the elimination of specific risk through diversification leads to reduce the weights of assets with high specific risk. In fact, the variance ratio reduces the weights of the assets with a total risk greater than the portfolio risk, while it increases the weights with a total risk lower than the one of the portfolio.

3. The mean variance ex-post optimal weight as a performance measure

In this section we explore the meaning and properties of the ex-post optimal weight as a performance measure in the mean-variance framework. Let us underline that we distinguish between two optimal weights: The ex-ante weights and the ex-post weights. Ex-ante weights are data that may come from a market index of from solving an optimization problem. Their role simply consists of defining the portfolio with which we are going to compare the ex-post optimal portfolio. We need these ex-ante weights mainly to incorporate alpha into the analysis. Since in the present context we have defined the Sharpe ratio as the objective function to be maximized, we take its ex-post value as the performance measure for the portfolio under analysis as a whole. The usual application of the Sharpe ratio as a performance measure combines the ex-post returns with the ex-ante weights. The performance measure that we propose for each stock in the portfolio consists of the optimal weights obtained by solving the Markowitz model for the actual data. Then, the proportion of each stock into the ex-post optimal portfolio is the answer to the question: Which capacity has generated the stock to create the optimal portfolio? The answer is the proportion of the stock into this portfolio. The remaining of this section is divided into relating the ex-post optimal weight to Jensen's alpha and into examining this weight from the point of view of performance axioms.

3.1. The ex-post optimal weight as a function of alpha

The ex-post optimal weight, i.e. the measure we propose, is compatible with Jensen's alpha as we show next. Jensen's alpha is calculated on basis of market risk premium and beta both obtained through the ex-ante, i.e. the actual, portfolio weights:

$$\alpha_i = \left(\bar{R}_i - r\right) - \left(\bar{R}_M^a - r\right)\beta_i^a \tag{20}$$

where $\bar{R}_M^a$ and β_i^a denote respectively the market rate of return and the beta coefficient of asset i both calculated through the ex-ante weights.

Taking into account (12) and (20) the asset's risk premium can be expressed through the ex-post market return and ex-post beta or through their ex-ante equivalents[4] and, then, alpha:

$$\bar{R}_i - r = \left(\bar{R}_M - r\right)\beta_i = \alpha_i + \left(\bar{R}_M^a - r\right)\beta_i^a \tag{21}$$

Thus, substituting this result into (13), the self-generated risk premium can be written as a function of alpha:

$$\bar{\pi}_i = \alpha_i + \left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M - r\right)\beta_{i\,j} \tag{22}$$

Subtracting and adding the product of the ex-ante risk premium by the ex-ante crossed beta , $\left(\bar{R}_M^a - r\right)\beta_{i\,j}^a$, to (22), we can write :

$$\bar{\pi}_i = \alpha_i + \left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a + \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a - \left(\bar{R}_M - r\right)\beta_{i\,j} \tag{23}$$

Hence, the self-generating risk premium can be expressed as the addition of three effects:

- Alpha effect: α_i .
- Crossed beta effect:

$$C\beta_i = \left[\left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\right] \tag{24}$$

- Weight changes effect:.

$$W_i = \left[\left(\bar{R}_M^a - r\right)\beta_{i\,j}^a - \left(\bar{R}_M - r\right)\beta_{i\,j}\right] \tag{25}$$

On this basis, we can say that the self-generated risk premium consists of the sum of alpha, the impact of the change from ordinary beta to crossed beta, and the impact of the differences due to the change from ex-ante weights, contained in $\bar{R}_M^a$ and $\beta_{i\,j}^a$, to the ex-post weights contained in $\bar{R}_M$ and $\beta_{i\,j}$. The crossed beta effect shows that the self-generated risk premium is reduced according to the contribution of the stock to the systematic risk of the other stocks in the portfolio. This contribution is measured by crossed beta that, as we have seen in (8) has on its numerator the weighted average of the covariances between the stock under analysis and the other stocks in the portfolio.

The weight changes effect measures the variation in the crossed risk premium due to the change of proportions in the ex-post portfolio

These effects can be incorporated into the optimal weight by substituting (23) in (14). Then, each effect is adjusted according to the variance ratio and the inverse of the market risk premium. The corresponding equation turns out to be:

$$x_i = \frac{\sigma_M^2}{\sigma_i^2}\left[\frac{\alpha_i}{\bar{R}_M - r} + \frac{\left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M^a - r\right)\beta_{ij}^a}{\bar{R}_M - r} + \frac{\left(\bar{R}_M^a - r\right)\beta_{ij}^a - \left(\bar{R}_M - r\right)\beta_{ij}}{\bar{R}_M - r}\right] \quad (26)$$

i.e.,

$$x_i = \frac{\sigma_M^2}{\sigma_i^2}\left[\frac{\alpha_i}{\bar{R}_M - r} + \frac{C\beta_i}{\bar{R}_M - r} + \frac{W_i}{\bar{R}_M - r}\right] \quad (27)$$

It stems from this result that to capture a large size of the market a stock must obtain a high alpha, a low crossed beta and a crossed ex-post required risk premium lower than the ex-ante one.

3.2. The coherence of the optimal weight as a performance measure

The coherence of the optimal weight as a performance measure requires that the model (mean-variance or mean- Gini in this paper) in which it is based fulfils the conditions that make it appropriate for the case under analysis. As stated, the optimal weight for mean-variance is based on the maximization of the Sharpe ratio of the optimal portfolio. In this respect, it must be taken into account that, as pointed out by Schuhmacher and Eling (2012) following Sinn (1983) and Meyer (1987), if the distribution of the random returns fulfils the location and scale property, then the mean-variance model fulfils as well the expected utility requirements. In order to be more precise, next we apply the axiomatic conditions that any performance measure should satisfy as presented by Schuhmacher and Eling (2012). According to these authors *an admissible performance measure is using admissible risk and reward measures* (Schuhmacher and Eling 2012, p. 2077).

Any admissible risk measure (u) fulfils the conditions of:

i. Positive homogeneity: $u\left(k\ A_i\right)=k\ u\left(A_i\right)\ \forall k$ and $u\left(0\right)=0$, and

ii. Functional translation invariance: $u\left(A_i+s\right)\leq u\left(A_i\right)$ for $s>0$ where s is a constant.

Any admissible reward measure (RW) must fulfil:

i. Positive homogeneity: $RW\left(k\ A_i\right)=k\ RW\left(A_i\right)$, and

ii. Functional translation invariance for reward measures,

$RW\left(A_i+s\right)\geq RW\left(A_i\right)$ for $s>0$.

Schuhmacher and Eling (2012, eq, 2 p. 2078) define the performance measure as the ratio between the reward and the risk measures. Now, it is convenient to express (3) as:

$$x_i=\frac{\dfrac{\left(\bar{R}_i-r\right)}{\left(\bar{R}_M-r\right)}\sigma_M^2-\sum\limits_{j=1\forall j\neq i}^{n}x_jCov\left(R_i,R_j\right)}{\sigma_i^2} \qquad (\ 28)$$

On this basis we can examine the applicability of the performance axioms to (28). We take the numerator as the reward measure adjusted to risk, and the denominator as the risk measure. The numerator fulfils the conditions of positive homogeneity and functional translation invariance. In effect:

i) Positive homogeneity in the numerator:

If we substitute $\left(R_i - r\right)$ by $k\ \left(R_i - r\right)$, we have $Cov\left(k\ R_i, R_j\right) = k\ Cov\left(R_i, R_j\right)$. Thus:

$$\frac{k\ \left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)} - \sum_{j=1 \forall j \neq i}^{n} \frac{x_j Cov\left(k\ R_i, R_j\right)}{\sigma_M^2} = k\left(\frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)} - \sum_{j=1 \forall j \neq i}^{n} \frac{x_j Cov\left(R_i, R_j\right)}{\sigma_M^2}\right) \tag{29}$$

which shows that the numerator fulfils positive homogeneity.

ii) Functional translation invariance in the numerator

Substituting R_i by $R_i + s$ in the numerator, we realize that it fulfils the condition of functional translation invariance for reward measures:

$$\frac{\bar{R}_i - r + s}{\bar{R}_M - r} - \sum_{j=1 \forall j \neq i}^{n} \frac{x_j Cov\left(\left(R_i + s - r\right), \left(R_j - r\right)\right)}{\sigma_M^2} = \frac{\left(\bar{R}_i - r\right) + s}{\left(\bar{R}_M - r\right)} - \sum_{j=1 \forall j \neq i}^{n} \frac{x_j Cov\left(R_i, R_j\right)}{\sigma_M^2}$$

(30)

Hence:

$$\frac{\left(\bar{R}_i - r\right) + s}{\left(\bar{R}_M - r\right)} - \sum_{j=1 \forall j \neq i}^{n} \frac{x_j Cov\left(R_i, R_j\right)}{\sigma_M^2} > \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)} - \sum_{j=1 \forall j \neq i}^{n} \frac{x_j Cov\left(R_i, R_j\right)}{\sigma_M^2} \tag{31}$$

The denominator, being a variance, fulfils as known the conditions of positive homogeneity and functional translation invariance, because $Var\left(k \cdot R_i\right) = k\sigma_i^2$, and $Var\left(R_i + s\right) = \sigma_i^2$.

All in all, the optimal weight as a performance measure is not universally applicable because it is subjected to the applicability of the optimization model from where it comes from. What the analysis of this Section has shown is that, in the contexts of mean-variance, the optimal weight is a coherent performance measure. This conclusion can be extended to the mean Gini paradigm as we argue after equation (61).

4. The Ginis' portfolio frontier approach

As know, Gini's Mean Difference (GMD) consists of a dispersion measure based on the absolute differences between two random variables or one random variable with itself (Yitzhaki and Schechtman 2013 p. 13)[5]. Denoting it by Δ and the two random variables by R_i and R_j, we can write:

$$\Delta = E\left\{\left|R_i - R_j\right|\right\} \tag{32}$$

One of the ways of expressing GMD (Shalit and Yitzhaki, 1984; Yitzhaki and Schechtman 2013 p. 18) consists of four times the covariance between the random variable (R) and its cumulative probability distribution $(F(R))$:

$$\Delta_j = 4Cov\left[R_j, F\left(R_j\right)\right] \tag{33}$$

or, for the crossed GMD, i.e. the co-Gini, with the cumulative distribution function of another variable:

$$\Delta_{ij} = 4Cov\left[R_i, F\left(R_j\right)\right] \tag{34}$$

The mean-Gini portfolio theory has a wider approach than the mean-variance paradigm (Shalit and Yitzhaki, 1984), because it is compatible with the second degree conditions of stochastic dominance (Yitzhaki and Schechtman, 2013, p. 389). For this reason, we next apply it to the performance analysis through the optimal weights of assets into a portfolio. In this context, in order to facilitate the optimization (Shalit and Yitzhaki, 1984; Yitzhaki and Schechtman 2013 p. 388), the risk measure $\left(\Delta^*\right)$ is taken as half of the GMD. Thus, the risks of asset i, portfolio M^* and asset i with respect to portfolio M^* are, respectively, expressed as[6]:

$$\Delta_i^* = 2Cov\left[R_i, F\left(R_i\right)\right] \tag{35}$$

$$\Delta^*_{M^*} = 2Cov\left[R_{M^*}, F\left(R_{M^*}\right)\right] \quad (36)$$

$$\Delta^*_{iM^*} = 2Cov\left[R_i, F\left(R_{M^*}\right)\right] \quad (37)$$

where M^* denotes now the optimal portfolio.

Then, the Gini's efficient frontier optimization problem consists of minimizing the portfolio GMD ($\Delta^*_{M^*}$) subject to a specified rate of return $\left(\sum_{i=1}^{n} x^*_i R_i = R^*\right)$ and to the budget restriction $\left(\sum_{i=1}^{n} x^*_i = 1\right)$. According to the approach of this article, we exclude the restriction that forbids short sales $\left(i.e.\ x^*_i \geq 0\right)$. We denote the optimal weight of asset i in the Gini optimal portfolio by x^*_i in order to distinguish it from its counterpart in the mean-variance paradigm (x_i). After the introduction of the risk free asset the optimization problem consists of maximizing the Sharpe ratio, now the Sharpe-Gini ratio $\left(SG\right)$:

$$SG_{M^*} = \frac{\sum_{j=1}^{n} x^*_j \left(\bar{R}_j - r\right)}{\Delta^*_{M^*}} \quad (38)$$

Our goal is to express the proportion of a stock into the optimal portfolio in such a way that in enables us to analyze it as a performance measure. To achieve this goal, we depart from the mean-Gini Security Market Line that Shalit and Yitzhaki (1984) express as:

$$\bar{R}_i = r + \left(\bar{R}_{M^*} - r\right) \frac{Cov\left[R_i, F\left(R_{M^*}\right)\right]}{Cov\left[R_{M^*}, F\left(R_{M^*}\right)\right]} \quad (39)$$

where, denoting by β^* the Gini-beta coefficient, following Yitzhaki and Schechtman (2013 p.46), we can write:

$$\beta_i^* = \frac{Cov\left[R_i, F\left(R_{M^*}\right)\right]}{Cov\left[R_{M^*}, F\left(R_{M^*}\right)\right]} \tag{40}$$

Hence:

$$\bar{R}_i = r + \left(\bar{R}_{M^*} - r\right)\beta_i^* \tag{41}$$

Let us multiply and divide the right hand side of (41) by the ordinary least squares beta between asset i and the Gini optimal portfolio that we will denote by $\beta_i^{'}$:

$$\beta_i^{'} = \frac{Cov\left(R_i, R_{M^*}\right)}{\sigma_{M^*}^2} \tag{42}$$

Then (41) becomes:

$$\bar{R}_i = r + \left(\bar{R}_{M^*} - r\right)\frac{Cov\left(R_i, R_{M^*}\right)}{\sigma_{M^*}^2}\frac{\sigma_{M^*}^2}{Cov\left(R_i, R_{M^*}\right)}\beta_i^* \tag{43}$$

Developing the expression of the covariance between R_i and R_{M^*}, we have:

$$Cov\left(R_i, R_{M^*}\right) = Cov\left(R_i, \sum_{j=1}^{n} x_j^* R_j\right) \tag{44}$$

Hence:

$$Cov\left(R_i, R_{M^*}\right) = x_i^*\sigma_i^2 + \sum_{j=1\forall j\neq i}^{n} x_j^* Cov\left(R_i, R_j\right) \tag{45}$$

Substituting (42) and (45) in (43), we have:

$$\bar{R}_i = r + \left(\bar{R}_{M^*} - r\right)\frac{x_i^*\sigma_i^2 + \sum_{j=1\forall j\neq i}^{n} x_j^* Cov\left(R_i, R_j\right)}{\sigma_{M^*}^2}\frac{\beta_i^*}{\beta_i^{'}} \tag{46}$$

Clearing x_i^* in this expression, we obtain:

$$x_i^* = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_{M^*} - r\right)} \frac{\sigma_{M^*}^2}{\sigma_i^2} \frac{\beta_i^{'}}{\beta_i^*} - \frac{1}{\sigma_i^2} \sum_{j=1 \forall j \neq i}^{n} x_j^* Cov\left(R_i, R_j\right) \qquad (47)$$

Recalling (4) and (5), we realize that the second addend of (45) is the crossed covariance $\left(CV_i^*\right)$ for the mean-Gini approach:

$$CV_i^* = \sum_{j=1 \forall j \neq i}^{n} x_j^* Cov\left(R_i, R_j\right) \qquad (48)$$

From where we can break down Gini's beta into its self-generated $\left(\beta_{i\,i}^{'}\right)$ and crossed $\left(\beta_{i\,j}^{'}\right)$ components:

$$\beta_i^{'} = \frac{Cov\left(R_i, R_{M^*}\right)}{\sigma_{M^*}^2} = \frac{x_i^* \sigma_i^2}{\sigma_{M^*}^2} + \frac{\sum_{j=1 \forall j \neq i}^{n} x_j^* Cov\left(R_i, R_j\right)}{\sigma_{M^*}^2} \qquad (49)$$

Hence:

$$\beta_{i\,i}^{'} = \frac{x_i^* \sigma_i^2}{\sigma_{M^*}^2} \qquad (50)$$

$$\beta_{i\,j}^{'} = \frac{\sum_{j=1 \forall j \neq i}^{n} x_j^* Cov\left(R_i, R_j\right)}{\sigma_{M^*}^2} \qquad (51)$$

$$\beta_i^{'} = \beta_{i\,i}^{'} + \beta_{i\,j}^{'} \qquad (52)$$

Taking (51) into account and multiplying and dividing the second addend of the right hand side of (47) by $\sigma_{M^*}^2$, we can express the optimal weight as a function of crossed beta through an equation that parallels (10):

$$x_i^* = \frac{\left(\bar{R}_i - r \big/ \bar{R}_{M^*} - r\right)\left(\frac{\beta_i^{'}}{\beta_i^*}\right) - \beta_{i\,j}^{'}}{\left(\sigma_i^2 \big/ \sigma_M^{*2}\right)} \qquad (53)$$

Multiplying and dividing $\beta^{'}_{i\,j}$ in this equation by $\left(\bar{R}_{M^*}-r\right)$and rearranging, we obtain:

$$x_i^* = \frac{\sigma^2_{M^*}}{\sigma^2_i} \frac{\left(\bar{R}_i - r\right)\left(\dfrac{\beta^{'}_i}{\beta^*_i}\right) - \left(\bar{R}_{M^*} - r\right)\beta^{'}_{i\,j}}{\left(\bar{R}_{M^*} - r\right)} \tag{54}$$

Next we introduce in the Gini context the concept of self-generated risk premium that we have defined in (13). From (41), we can express the expected risk premium of asset i as:

$$\bar{R}_i - r = \left(\bar{R}_{M^*} - r\right)\beta^{'}_i \frac{\beta^*_i}{\beta^{'}_i} \tag{55}$$

Substituting $\beta^{'}_i$ according to (52):

$$\bar{R}_i - r = +\left(\bar{R}_{M^*} - r\right)\left(\beta^{'}_{i\,i} + \beta^{'}_{i\,j}\right)\frac{\beta^*_i}{\beta^{'}_i} \tag{56}$$

This expression enables us to break down the asset risk premium into its self-generated and crossed parts:

$$\bar{R}_i - r = \left(\bar{R}_{M^*} - r\right)\beta^{'}_{i\,i} \frac{\beta^*_i}{\beta^{'}_i} + \left(\bar{R}_{M^*} - r\right)\beta^{'}_{i\,j} \frac{\beta^*_i}{\beta^{'}_i} \tag{57}$$

Thus, the expected self-generated risk premium turns out to be:

$$\bar{\pi}^*_i = \left(\bar{R}_{M^*} - r\right)\beta^{'}_{i\,i} \frac{\beta^*_i}{\beta^{'}_i} \tag{58}$$

or what it is equivalent:

$$\bar{\pi}^*_i = \left(\bar{R}_i - r\right) - \left(\bar{R}_{M^*} - r\right)\beta^{'}_{i\,j}\left(\frac{\beta^{'}_i}{\beta^*_i}\right) \tag{59}$$

Then the variance adjusted self-generated risk premium can be written as:

$$\hat{\pi}^*_i = \frac{\sigma^2_{M^*}}{\sigma^2_i}\bar{\pi}^*_i \tag{60}$$

On this basis, the optimal proportion becomes:

$$x_i^* = \frac{\frac{\sigma_{M^*}^2}{\sigma_i^2}\bar{\pi}_i^*}{\bar{R}_{M^*} - r} = \frac{\hat{\pi}_i^*}{\bar{R}_{M^*} - r} \tag{61}$$

The connection between alpha and the self-generated risk premium in the mean-Gini approach is similar to the one we have presented for the mean-variance case because the unique change is the beta correction. For the same reason, its relationship with the performance axioms also parallels the analysis we have developed in the subsection 3.2. If we take the same ex-ante portfolio that we have taken for the mean-variance case, alpha does not change and equation (21) is still valid. In fact, the ex-ante portfolio is not necessarily the outcome of an optimization process. Instead, it is a given portfolio, which ex-post performance we can analyze from the mean-variance of mean-Gini-viewpoints. Thus, substituting the expected[7] asset risk premium in the expression of Gini self-generated risk premium, i.e. in equation (59), we have:

$$\pi_i^* = \left[\alpha_i + \left(\bar{R}_M^a - r\right)\beta_i^a\right]\left(\frac{\beta_i^{'}}{\beta_i^*}\right) - \left(\bar{R}_{M^*} - r\right)\beta_{i\,j}^{'} \tag{62}$$

which is equivalent to (22) after introducing the beta correction.

Subtracting and adding $\left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\left(\beta_i^{'}/\beta_i^*\right)$ to (62), we can express the Gini self-generated risk premium as the addition of the three effects that we have identified in (23), now with the beta correction:

$$\bar{\pi}_i^* = \alpha_i\left(\frac{\beta_i^{'}}{\beta_i^*}\right) + \left[\left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\right]\left(\frac{\beta_i^{'}}{\beta_i^*}\right) + \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\left(\frac{\beta_i^{'}}{\beta_i^*}\right) - \left(\bar{R}_{M^*} - r\right)\beta_{i\,j}^*$$

(63)

Thus, the Gini self-generated risk premium can be expressed as the addition of three effects:

- Gini Alpha effect:

$$G\alpha_i = \alpha_i\left(\frac{\beta_i^{'}}{\beta_i^{*}}\right) \tag{64}$$

- Gini Crossed Beta effect:

$$G\beta_i = \left[\left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\right]\left(\frac{\beta_i^{'}}{\beta_i^{*}}\right) \tag{65}$$

- Gini weight changes effect:

$$GW_i = \left[\left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\left(\frac{\beta_i^{'}}{\beta_i^{*}}\right) - \left(\bar{R}_{M^*} - r\right)\beta_{i\,j}^{*}\right] \tag{66}$$

Substituting (63) in (61), the optimal weight is expressed as a function of the three effects we have identified. Each one of them becomes adjusted by the variance ratio and the inverse of the market risk premium according to the following equation:

$$x_i^* = \frac{\sigma_M^{*2}}{\sigma_i^2}\,\frac{\alpha_i\left(\frac{\beta_i^{'}}{\beta_i^{*}}\right) + \left[\left(\bar{R}_M^a - r\right)\beta_i^a - \left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\right]\left(\frac{\beta_i^{'}}{\beta_i^{*}}\right) + \left[\left(\bar{R}_M^a - r\right)\beta_{i\,j}^a\left(\frac{\beta_i^{'}}{\beta_i^{*}}\right) - \left(\bar{R}_{M^*} - r\right)\beta_{i\,j}^{*}\right]}{\bar{R}_M^* - r} \tag{67}$$

i.e.:

$$x_i^* = \frac{\sigma_M^{*2}}{\sigma_i^2}\left[\frac{G\alpha_i}{\bar{R}_{M^*} - r} + \frac{G\beta_i}{\bar{R}_{M^*} - r} + \frac{GW_i}{\bar{R}_{M^*} - r}\right] \tag{68}$$

The interpretation of the optimal weight as the addition the impacts of the three effects is similar to the one of the mean-variance case.

5. Empirical illustration

This empirical illustration is based on the corporations that constituted the DJIA in 2013 and generated a positive risk premium in that year. The basic data are the weekly rates of return on the DJIA stocks calculated from the corresponding adjusted values downloaded from Yahoo! Finance. Table 1 shows the basic statistics of these stocks. Theoretically, the stocks with negative risk premia could have been included in the weight performance analysis. Nevertheless, strongly negative rates often generate high short positions on these stocks that, after the introduction of the risk free asset, lead to a market portfolio in a short position[8]. To avoid centring the analysis on this peculiar situation, we have excluded the four stocks with negative risk premia in 2013 and have developed the study for the remaining twenty six. All stocks are denoted by their codes.

INSERT TABLE 1

In Table 2 we show the results that stem from the application of the mean-variance analysis. Row 1 displays the optimal weights, i.e. the positions in each stock that would have maximized the ex-post Sharpe ratio. Rows 2-8 show how these weights have been generated. In rows 3-5 we break down beta into its self-generated and crossed parts. The negative self-generated betas that we observe in row 3 are due to the negative positions of their corresponding stocks, as it stems from equation (7). All betas are lower than 1, although, due to the effect of short positions, their weighted average is, of course, 1. Row 6 shows the self-generated risk premium obtained from the product between the market risk premium and self-generated beta, while can also be obtained by subtracting the part of the required rate of return due to crossed beta from the asset risk premium. Row 7 shows the variance ratio. The self-generated risk premium and the variance ratio can be regarded as the two central variables in the determination of the optimal weight. In effect, their product leads to the variance adjusted self-generated risk premium, whose quotient with the market risk premium turns out to be the optimal weight, that we repeat in row 10 in order to underline how it is obtained. The market risk premium, obviously a constant for all stocks, simply introduces a change of scale.

Rows 11-15 can be regarded as the step that links the drivers of the optimal weights to the identification of the parts of them that are due to the effects of alpha, crossed beta and weight changes shown in rows 16-18. This intermediate step shows how the self generating risk premium is broken down into the impacts of these three effects on it. The sum of rows 11-14 equates row 15, where we repeat the self-generated risk premium in order to underline this property.

The composition of the optimal weights, displayed in rows 16-18, stresses their analytical capacity as a performance measure by showing how each weight can be expressed through the parts of it that can be attributed to alpha, crossed beta and weight changes. Capturing an abnormal return, i.e. alpha, increases the weight, but the crossed beta also has a relevant role on the weight. The lower the impact of the stock on the systematic risk of other assets, namely the lower the crossed beta, the greater the weight, as expresses the crossed beta effect. In addition, the lower the ex-post crossed risk premium with respect to its ex-ante equivalent, the greater the weight as well. The variance ratio multiplies each one of these three effects, as shown in equation (27).

Let us compare four stocks: JNJ, MSFT, BAC and XOM. MSFT has captured a greater alpha than JNJ (0.30% vs. 0.09%, both monthly). Nevertheless, the weight changes effect is substantially lower for MSFT than for JNJ (0.38% vs. 0.57%), while their crossed beta effects are similar (0.096 for JNJ and 0.08 fo4 MSFT). In addition, the variance ratio has opposite effects on both stocks, because it reduces the impacts of alpha, crossed beta and proportion changes for MSFT (39.26%) and increases them for JNJ (189.78%). The final outcome is an ex-post optimal weight of 61.58% for JNJ, while MSFT does not surpass the 13.70%. Focusing on BAC and XOM, both in short position, we observe that both show a similar negative alpha (-0.10% for BAC and -0.11% for XOM). The weight changes effect is substantially lower for BAC than for XOM (-0.79% vs. -0.54%). The crossed beta effect improves the position of BAC (0.20%) and only has a slight impact on XOM (0.04%). Nevertheless, the variance ratio reduces approximately to their 50% the effects on BAC and doubles the effects on XOM, with final weights of -15.76% for BAC and -57.31% for XOM. Although the signs of alpha and weight changes usually coincide, in some cases the sign of weight changes turns a position that would be positive for alpha into a negative one and vice-versa. These are the cases of DD in which a negative weight change dominates a positive alpha (-0.77% vs. 0.20%), and PG in which a positive weight change dominates a negative alpha (0.65% vs. -0.09%).

INSERT TABLE 2

Table 3 shows the results for the Gini optimal portfolio analysis. Comparing the Gini weights with the mean-variance weights, we observe that for many corporations the percentages are close, but in some they are rather different. Gini weights have been obtained through a twofold procedure that has led to the same results: The covariances method through *Excel* and the permutations method through *Mathematica*. We also compare (rows 6 and 7) the actual risk premia with the risk premia obtained through the Security Market Line for the Gini portfolio optimization. Theoretically, both results should be equal, but, in fact, there are some small differences that we attribute to the numerical optimization procedures. The analysis has been developed for the SML risk premia. The analysis for the mean-Gini framework parallels the case of the mean variance, although, as expected, the numerical values are different. A case in point is the comparison between DIS and MCD. The position of the former changes from long to short (6.36% to -14.10%) when we switch to mean-Gini, while the position of the latter shows the opposite change (-5.54% to 10.04%). DIS' alpha is 0.10%, but changes to 0.06% after the Gini's beta correction. The crossed beta changes from 0.08% to 0.04%, but the main change comes from the weight change that evolves from a nil value for mean variance to a -0.42% for mean-Gini. The addition of these effects leads the self-generated risk premium to move from 0.18% for mean-variance to -0.32% for mean-Gini. Conversely, MCD's negative alpha (-0.20%) is not compensated by the remaining two effects in the main-variance case, but this compensation effectively takes place for mean-Gini: the beta correction produces a small reduction in alpha which becomes a -0.17%, but the real cause of the sign change is the weight change effect that achieves 0.22% for mean-Gini while it is only 0.09% for mean-variance.

To sum up, from the ex-post viewpoint, each optimal weight is generated by the interaction between the self-generated risk premium and the variance ratio. Their product, expressed per each point of the market risk premium, enables us to obtain the optimal weights. The introduction of the ex-ante viewpoint enlarges the analysis by showing the parts of each optimal weight that can be attributed to alpha, crossed beta, and to the changes of ex-post weights with respect to ex-ante weights.

INSERT TABLE 3

Figure 1 shows the plots for the mean-variance and Gini cases.

INSERT FIGURE 1

6. Conclusions

Let us recall the research question about the performance of common stocks that has motivated this article: How does the market regard the competitiveness of these stocks with respect to other stocks in the market? Can we find a quantitative measure for this competitiveness? Our paper shows that the optimal weight from the ex-post data answers this question. Our proposal is based on the fact that the optimal weight expresses the budget's proportion which, invested in the asset, better contributes to the maximization of the portfolio performance. In other words, it expresses the asset's capacity for generating an optimal value of the objective function of the portfolio selection model that is being used. Thus, a larger weight means a larger contribution to the overall goal. It also means a larger size in the ideal market of the optimal ex-post portfolio. This proposition is, in itself, independent from the objective function chosen for portfolio selection, although we have developed its analysis for the mean-variance and mean-Gini paradigms

The optimal weight not only is, in itself, a performance measure, but also contains very relevant financial information for managers and investors. In effect, it is the outcome of the interaction between the variance ratio and the self-generated risk premium. The latter is central for understanding the meaning of the optimal weight. It stems from breaking down beta into self-generated beta and crossed beta, which, respectively, express the parts of the systematic risk due to the stock itself (self-generated) and its interaction with the rest of the stocks in the portfolio (crossed). Furthermore, the analysis of the self-generated risk premium shows that it can be expressed as the addition of three effects: alpha, crossed beta and the impact of the overall weight changes from the ex-ante to the ex-post optimal portfolio. Thus, this result connects the optimal weight with alpha and the new risk decomposition we have found out. As stated, for a manager who seeks a good performance from the optimal weight perspective a high alpha is not enough because it interacts with crossed beta and the overall weight changes. All three effects must be taken into account. The condition that justifies a long position on an asset is, as equation (18) shows, an actual risk premium greater than the crossed risk premium. This condition can be regarded as a minimum managerial goal for value creation. As stated, this condition is softer than obtaining a Treynor ratio that outperforms the market risk premium. From the perspective of the optimal ex-post portfolio, stocks can be grouped in gainers and losers according to their long or short position in the ex-post optimal portfolio. A limitation to the practical applicability of this model lies in securities with ex-post negative risk premium. The problem of including them is that, in this case, short positions may dominate the ex-post optimal portfolio even leading to a short position of the optimal portfolio itself, as we have stated in the empirical study of Section 5.

The empirical illustration we have developed through the mean-variance and mean-Gini approaches shows the embedded information in the stocks optimal weights, which includes the condition for being in long or short positions, i.e., the breakeven point for a stock to be in one or the other position. Mostly, the positions that stem from both analyses are quite similar, but in case of a relevant difference between them the Gini result should prevail due to its more complete appraisal of the random properties of the stocks under analysis.

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APPENDIX

A.1 Obtaining the optimal weight from the Sharpe ratio

The Sharpe ratio of the market portfolio consists of:

$$S_M = \frac{\bar{R}_M - r}{\sigma_M} \qquad \text{(A1)}$$

where:

$$\bar{R}_M - r = x_i\left(\bar{R}_i - r\right) + \sum_{\substack{j=1\\ \forall j \neq i}}^{n} x_j\left(\bar{R}_j - r\right) \qquad \text{(A2)}$$

$$\sigma_M = \left(x_i^2\sigma_i^2 + \sum_{j=1 \forall j\neq i}^{n} x_j^2\sigma_j^2 + \sum_{i=1}^{n}\sum_{j=1 \forall j\neq i}^{n} x_i x_j Cov\left(R_i, R_j\right) + \sum_{\forall\left(\substack{j, j'\neq i\\ j\neq j'}\right)}\sum x_j x_{j'} Cov\left(R_j, R_{j'}\right)\right)^{1/2}$$

(A3)

Let us derive the Sharpe ratio with respect to x_i:

$$\frac{dS_M}{dx_i} = \frac{\left(\bar{R}_i - r\right)\sigma_M + \left(\bar{R}_M - r\right)\frac{d\sigma_M}{dx_i}}{\sigma_M^2} \qquad \text{(A4)}$$

$$\frac{d\sigma_M}{dx_i} = \frac{1}{2}\left(2x_i\sigma_i^2 + 2\sum_{\forall j\neq i} x_j\sigma_{ij}\right)\sigma_M^{-1} = \frac{x_i\sigma_i^2 + \sum_{\forall j\neq i} x_j\sigma_{ij}}{\sigma_M} \qquad \text{(A5)}$$

$$\frac{dS_M}{dx_i} = \frac{\left(\bar{R}_i - r\right)}{\sigma_M} - \frac{\left(\bar{R}_M - r\right)\left(x_i\sigma_i^2 + \sum_{\forall j\neq i} x_j\sigma_{ij}\right)\frac{1}{\sigma_M}}{\sigma_M^2} \qquad \text{(A6)}$$

Equating dS_M / dx_i to zero, we have:

$$\frac{\left(\bar{R}_M - r\right)\left(x_i\sigma_i^2 + \sum_{\forall j\neq i} x_j\sigma_{ij}\right)}{\sigma_M^3} = \frac{\left(\bar{R}_i - r\right)}{\sigma_M} \qquad \text{(A7)}$$

Hence:

$$\left(x_i\sigma_i^2 + \sum_{\forall j\neq i} x_j\sigma_{i\,j} \right) = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)}\sigma_M^2 \tag{A8}$$

Clearing x_i we obtain:

$$x_i = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)}\frac{\sigma_M^2}{\sigma_i^2} - \frac{\sum_{\forall j\neq i} x_j\sigma_{i\,j}}{\sigma_i^2} \tag{A9}$$

A.2 Obtaining the optimal weight from the Security Market Line

The optimal weight of a stock in the market portfolio can also be obtained after performing some operations on the Security Market Line. Let us recall that its equation (Sharpe 1964) is:

$$\bar{R}_i = r + \left(\bar{R}_M - r\right)\frac{Cov\left(R_i, R_M\right)}{\sigma_M^2} \tag{A10}$$

In order to make explicit the weight of asset i in the market portfolio, we write their covariance as:

$$Cov\left(R_i, R_M\right) = Cov\left(R_i, \sum_{j=1}^{n} x_j R_j \right) \tag{A11}$$

Thus:

$$Cov\left(R_i, R_M\right) = x_i\sigma_i^2 + \sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i, R_j\right) \tag{A12}$$

Substituting (A12) in (A10) we have:

$$\bar{R}_i = r + \left(\bar{R}_M - r\right)\frac{x_i\sigma_i^2 + \sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i, R_j\right)}{\sigma_M^2} \tag{A13}$$

and clearing x_i in this equation, we obtain:

$$x_i = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)}\frac{\sigma_M^2}{\sigma_i^2} - \sum_{j=1\forall j\neq i}^{n} x_j \frac{Cov\left(R_i, R_j\right)}{\sigma_i^2} \tag{A14}$$

[1] Merton (1972), and Huang and Litzenberger (1988) show the detailed steps to follow to solve this problem.

[2] The expected risk premium becomes the average risk premium when we switch to ex-post optimization with a performance analysis purpose.

[3] These properties hold for the expected self-generated risk premium in ex-ante optimization and for the average risk premium obtained in the ex-post optimization with the performance purposes that we develop in this paper.

[4] To be more precise, both β_i and β_i^a are obtained from the ex-post returns. Nevertheless, the weights used in their calculations are the ex-ante ones for β_i^a and the ex-post ones for β_i.

[5] We substitute X_1 and X_2 in Yitzhaki and Schechtman by R_i and R_j .

[6] We adopt the notations used in Yitzhaki and Schechtman (2013).

[7] The same considerations about expected vs. average risk premium se have made in footnote 3 hold for the Gini case. For instance, $\bar{\pi}$ denotes the expected self-generated risk premium from the ex-ante perspective and the average self-generated risk premium from the ex-post perspective.

[8] The case of a market portfolio in short position is analysed by Huang and Litzenberger (1988, p. 79).

Table 1: BASIC DATA

Corporation	**AA**	**BA**	**BAC**	**CSCO**	**CVX**	**DD**	**DIS**	**GE**	**HD**	**INTL**	**JNJ**	**JPM**	**KO**	**MCD**	**MMM**	**MRK**	**MSFT**	**PFE**	**PG**	**T**	**TRV**	**UNH**	**UTX**	**VZ**	**WMT**	**XOM**
Mean	0.003	0.011	0.006	0.002	0.003	0.007	0.008	0.006	0.005	0.005	0.005	0.005	0.002	0.002	0.008	0.004	0.007	0.004	0.004	0.001	0.004	0.007	0.006	0.003	0.003	0.003
Variance	0.001	0.001	0.001	0.001	0.000	0.000	0.001	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000
St dev	0.029	0.024	0.031	0.035	0.018	0.021	0.025	0.020	0.023	0.023	0.016	0.023	0.020	0.016	0.018	0.019	0.035	0.022	0.020	0.020	0.017	0.029	0.020	0.022	0.017	0.015
Skewness	0.195	0.508	0.198	0.610	-0.188	0.423	0.375	-1.038	0.165	-0.282	-1.036	0.014	-0.484	-0.496	0.900	-0.211	-0.839	-0.344	-0.794	-0.259	-0.350	-0.436	-0.336	-0.066	-0.452	0.060
Kurtosis	0.409	-0.403	-0.773	4.003	-0.604	1.292	0.334	4.095	-0.196	-0.999	0.736	-0.226	0.057	-0.478	4.604	0.153	3.701	1.511	1.766	-0.047	0.018	1.406	-0.085	0.430	-0.584	-0.088
Jarque Bera	0.693	2.587	1.633	37.946	1.098	5.167	1.459	45.668	0.321	2.851	10.474	0.112	2.036	2.630	52.938	0.438	35.779	5.972	12.230	0.586	1.063	5.926	0.992	0.438	2.507	0.048
Correlation with M	0.086	0.458	0.180	0.052	0.150	0.321	0.291	0.284	0.221	0.181	0.319	0.227	0.079	0.101	0.420	0.192	0.185	0.153	0.156	0.018	0.220	0.245	0.277	0.098	0.151	0.151

TABLE 2: MEAN-VARIANCE OPTIMAL WEIGHT ANALYSIS

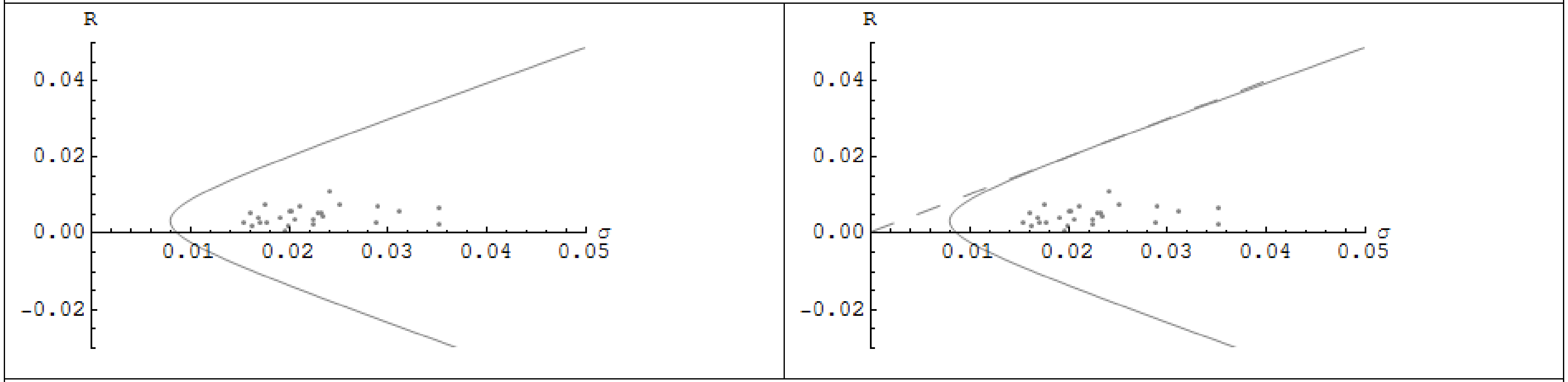
Row		Equation	AA	BA	BAC	CSCO	CVX	DD	DIS	GE	HD	INTL	JNJ	JPM	KO	MCD	MMM	MRK	MSFT	PFE	PG	T	TRV	UNH	UTX	VZ	WMT	XOM
1	**Optimal weights from CAPM**		-15.22%	72.61%	-15.76%	-33.14%	-16.35%	-35.54%	6.36%	41.50%	15.03%	16.19%	61.58%	6.64%	-43.14%	-5.54%	118.27%	-6.23%	13.70%	2.07%	33.23%	-129.13%	-29.91%	34.21%	4.78%	41.24%	19.85%	-57.31%
	Drivers of the optimal weights																											
2	Risk premium		0.0024	0.0108	0.0055	0.0018	0.0026	0.0066	0.0072	0.0056	0.0050	0.0042	0.0050	0.0051	0.0015	0.0016	0.0072	0.0036	0.0064	0.0034	0.0031	0.0004	0.0036	0.0070	0.0055	0.0022	0.0025	0.0023
3	Self-generated beta	(7)	-0.2589	0.8703	-0.3163	-0.8448	-0.1056	-0.3256	0.0828	0.3463	0.1676	0.1845	0.3245	0.0715	-0.3531	-0.0301	0.7495	-0.0467	0.3491	0.0215	0.2882	-1.0258	-0.1746	0.5925	0.0406	0.4278	0.1201	-0.2830
4	Crossed beta	(8)	0.3709	-0.3688	0.5715	0.9277	0.2258	0.6325	0.2498	-0.0871	0.0653	0.0083	-0.0932	0.1640	0.4244	0.1049	-0.4155	0.2126	-0.0531	0.1343	-0.1432	1.0423	0.3431	-0.2704	0.2143	-0.3278	-0.0030	0.3890
5	Beta[(3)+(4)]		0.1120	0.5015	0.2552	0.0829	0.1202	0.3069	0.3326	0.2592	0.2329	0.1928	0.2313	0.2355	0.0712	0.0748	0.3340	0.1660	0.2959	0.1557	0.1451	0.0165	0.1685	0.3220	0.2548	0.1000	0.1172	0.1059
6	Self-generated risk premium	(13)	-0.0056	0.0188	-0.0068	-0.0182	-0.0023	-0.0070	0.0018	0.0075	0.0036	0.0040	0.0070	0.0015	-0.0076	-0.0007	0.0162	-0.0010	0.0075	0.0005	0.0062	-0.0222	-0.0038	0.0128	0.0009	0.0092	0.0026	-0.0061
7	Variance ratio		0.5880	0.8344	0.4982	0.3923	1.5480	1.0914	0.7683	1.1983	0.8967	0.8775	1.8978	0.9291	1.2217	1.8379	1.5781	1.3355	0.3926	0.9660	1.1528	1.2588	1.7127	0.5774	1.1786	0.9640	1.6523	2.0250
8	Var-adj self gen risk premium	(15)	-0.0033	0.0157	-0.0034	-0.0072	-0.0035	-0.0077	0.0014	0.0090	0.0032	0.0035	0.0133	0.0014	-0.0093	-0.0012	0.0255	-0.0013	0.0030	0.0004	0.0072	-0.0279	-0.0065	0.0074	0.0010	0.0089	0.0043	-0.0124
9	Break even risk premium	(18)	0.0080	-0.0080	0.0123	0.0200	0.0049	0.0137	0.0054	-0.0019	0.0014	0.0002	-0.0020	0.0035	0.0092	0.0023	-0.0090	0.0046	-0.0011	0.0029	-0.0031	0.0225	0.0074	-0.0058	0.0046	-0.0071	-0.0001	0.0084
10	Optimal weight	(16)	-15.22%	72.61%	-15.76%	-33.14%	-16.35%	-35.54%	6.36%	41.50%	15.03%	16.19%	61.58%	6.64%	-43.14%	-5.54%	118.27%	-6.23%	13.70%	2.07%	33.23%	-129.13%	-29.91%	34.21%	4.78%	41.24%	19.85%	-57.31%
	The ex-post weights from the perspective of the ex-ante weights																											
11	Adjusted weights ex-ante (DJIA)		0.49%	0.89%	6.85%	1.49%	7.64%	3.62%	4.10%	1.47%	4.62%	1.44%	5.45%	3.24%	2.38%	5.99%	7.29%	2.94%	2.03%	1.75%	4.87%	2.11%	5.14%	4.58%	6.67%	2.94%	4.57%	5.45%
12	Alpha		-0.0013	0.0069	-0.0010	-0.0053	-0.0016	0.0020	0.0010	0.0013	0.0004	0.0015	0.0009	0.0004	-0.0019	-0.0020	0.0026	0.0014	0.0030	-0.0003	-0.0009	-0.0035	-0.0012	0.0034	-0.0004	-0.0016	0.0021	-0.0011
13	Crossed beta effect	(24)	0.0001	0.0002	0.0020	0.0006	0.0007	0.0005	0.0008	0.0002	0.0008	0.0002	0.0004	0.0005	0.0003	0.0005	0.0007	0.0003	0.0008	0.0003	0.0006	0.0002	0.0004	0.0012	0.0008	0.0004	0.0004	0.0004
14	Weight changes effect	(25)	-0.0044	0.0117	-0.0079	-0.0135	-0.0014	-0.0095	0.0000	0.0060	0.0025	0.0022	0.0057	0.0006	-0.0060	0.0009	0.0129	-0.0027	0.0038	0.0005	0.0065	-0.0189	-0.0030	0.0082	0.0004	0.0104	0.0001	-0.0054
15	Self generated risk premium	(23)	-0.0056	0.0188	-0.0068	-0.0182	-0.0023	-0.0070	0.0018	0.0075	0.0036	0.0040	0.0070	0.0015	-0.0076	-0.0007	0.0162	-0.0010	0.0075	0.0005	0.0062	-0.0222	-0.0038	0.0128	0.0009	0.0092	0.0026	-0.0061
	Optimal weight composition																											
16	Alpha		-3.67%	26.82%	-2.21%	-9.63%	-11.76%	10.15%	3.64%	7.12%	1.59%	6.19%	7.95%	1.91%	-10.64%	-17.35%	19.01%	8.67%	5.46%	-1.35%	-4.79%	-20.58%	-9.41%	9.22%	-2.02%	-7.23%	15.75%	-10.17%
17	Crossed beta		0.33%	0.60%	4.63%	1.01%	5.17%	2.45%	2.78%	0.99%	3.13%	0.98%	3.68%	2.19%	1.61%	4.05%	4.93%	1.99%	1.38%	1.18%	3.29%	1.43%	3.48%	3.10%	4.51%	1.99%	3.09%	3.68%
18	Weight changes		-11.88%	45.19%	-18.19%	-24.52%	-9.76%	-48.14%	-0.05%	33.39%	10.32%	9.03%	49.95%	2.53%	-34.11%	7.76%	94.34%	-16.89%	6.87%	2.24%	34.73%	-109.98%	-23.97%	21.89%	2.30%	46.47%	1.01%	-50.83%
19	Addition (optimal weight)		-15.22%	72.61%	-15.76%	-33.14%	-16.35%	-35.54%	6.36%	41.50%	15.03%	16.19%	61.58%	6.64%	-43.14%	-5.54%	118.27%	-6.23%	13.70%	2.07%	33.23%	-129.13%	-29.91%	34.21%	4.78%	41.24%	19.85%	-57.31%
			Portfolio risk premium		0.021595			Portfolio variance				0.000484																

TABLE 3: GINI PORTFOLIO ANALYSIS

Row	Corporation	Equation	AA	BA	BAC	CSCO	CVX	DD	DIS	GE	HD	INTL	JNJ	JPM	KO	MCD	MMM	MRK	MSFT	PFE	PG	T	TRV	UNH	UTX	VZ	WMT	XOM
1	Weights for Gini	(61)	-14.35%	84.91%	-24.85%	-33.41%	-35.69%	-43.52%	-14.10%	40.65%	27.84%	14.62%	68.09%	13.52%	-44.88%	10.04%	132.91%	-15.32%	11.97%	-17.55%	66.54%	-176.18%	-14.30%	43.15%	19.49%	70.45%	34.47%	-104.49%
	Drivers of the optimal weights																											
2	Gini mean difference	(35)	0.0161	0.0137	0.0180	0.0184	0.0102	0.0116	0.0142	0.0108	0.0132	0.0136	0.0086	0.0130	0.0113	0.0093	0.0092	0.0108	0.0183	0.0121	0.0112	0.0112	0.0095	0.0157	0.0115	0.0126	0.0098	0.0088
3	Beta OLS Gini weights		-0.0098	0.3621	0.1338	0.0695	0.0266	0.1751	0.1634	0.1262	0.1750	0.0743	0.2092	0.1860	0.0789	0.0626	0.2335	0.0824	0.2057	0.0966	0.1395	0.0177	0.1244	0.2559	0.2112	0.0637	0.1406	-0.0124
4	Beta Gini	(40)	0.1077	0.4523	0.2312	0.0784	0.1154	0.2856	0.3005	0.2412	0.2090	0.1723	0.2115	0.2146	0.0668	0.0731	0.2998	0.1447	0.2455	0.1366	0.1335	0.0130	0.1573	0.2903	0.2315	0.0895	0.1081	0.1002
5	Beta correction (row 3/ row 4)		-0.0907	0.8006	0.5785	0.8865	0.2308	0.6130	0.5437	0.5232	0.8376	0.4311	0.9890	0.8667	1.1821	0.8563	0.7788	0.5695	0.8382	0.7075	1.0451	1.3648	0.7903	0.8814	0.9124	0.7117	1.3005	-0.1239
6	Actual asset risk premium		0.0024	0.0108	0.0055	0.0018	0.0026	0.0066	0.0072	0.0056	0.0050	0.0042	0.0050	0.0051	0.0015	0.0016	0.0072	0.0036	0.0064	0.0034	0.0031	0.0004	0.0036	0.0070	0.0055	0.0022	0.0025	0.0023
7	Asset risk premium Gini SML	(41)	0.0026	0.0109	0.0056	0.0019	0.0028	0.0069	0.0072	0.0058	0.0050	0.0042	0.0051	0.0052	0.0016	0.0018	0.0072	0.0035	0.0059	0.0033	0.0032	0.0003	0.0038	0.0070	0.0056	0.0022	0.0026	0.0024
8	Variance ratio (Gini portfolio/asset)		0.8162	1.1583	0.6916	0.5446	2.1490	1.5150	1.0665	1.6634	1.2449	1.2182	2.6345	1.2898	1.6959	2.5514	2.1908	1.8539	0.5449	1.3411	1.6004	1.7475	2.3775	0.8016	1.6361	1.3382	2.2938	2.8112
9	Self generated OSL Gini beta	(50)	-0.1759	0.7330	-0.3593	-0.6134	-0.1661	-0.2873	-0.1322	0.2444	0.2236	0.1200	0.2585	0.1048	-0.2646	0.0394	0.6067	-0.0826	0.2196	-0.1309	0.4158	-1.0082	-0.0602	0.5384	0.1191	0.5264	0.1503	-0.3717
10	Crossed OSL Gini beta	(51)	0.1661	-0.3709	0.4930	0.6829	0.1927	0.4624	0.2955	-0.1182	-0.0486	-0.0457	-0.0493	0.0812	0.3435	0.0232	-0.3732	0.1650	-0.0138	0.2275	-0.2763	1.0259	0.1845	-0.2824	0.0921	-0.4628	-0.0096	0.3593
11	Gini self generated risk premium	(59)	-0.0042	0.0177	-0.0087	-0.0148	-0.0040	-0.0069	-0.0032	0.0059	0.0054	0.0029	0.0062	0.0025	-0.0064	0.0009	0.0146	-0.0020	0.0053	-0.0032	0.0100	-0.0243	-0.0015	0.0130	0.0029	0.0127	0.0036	-0.0090
12	Var adj Gini self gen risk premium	(60)	-0.0035	0.0205	-0.0060	-0.0081	-0.0086	-0.0105	-0.0034	0.0098	0.0067	0.0035	0.0164	0.0033	-0.0108	0.0024	0.0321	-0.0037	0.0029	-0.0042	0.0161	-0.0425	-0.0035	0.0104	0.0047	0.0170	0.0083	-0.0252
13	Break even risk premium		-0.0442	-0.0112	0.0206	0.0186	0.0201	0.0182	0.0131	-0.0055	-0.0014	-0.0026	-0.0012	0.0023	0.0070	0.0007	-0.0116	0.0070	-0.0004	0.0078	-0.0064	0.0181	0.0056	-0.0077	0.0024	-0.0157	-0.0002	-0.0700
	Ex-post weights from the perspective of the ex-ante weights																											
14	Alpha		-0.0013	0.0069	-0.0010	-0.0053	-0.0016	0.0020	0.0010	0.0013	0.0004	0.0015	0.0009	0.0004	-0.0019	-0.0020	0.0026	0.0014	0.0030	-0.0003	-0.0009	-0.0035	-0.0012	0.0034	-0.0004	-0.0016	0.0021	-0.0011
15	Alpha Gini effect	(64)	0.0001	0.0056	-0.0006	-0.0047	-0.0004	0.0012	0.0006	0.0007	0.0003	0.0007	0.0009	0.0004	-0.0022	-0.0017	0.0020	0.0008	0.0025	-0.0002	-0.0009	-0.0048	-0.0009	0.0030	-0.0003	-0.0012	0.0027	0.0001
16	Crossed beta effect	(65)	0.0000	0.0001	0.0012	0.0005	0.0002	0.0003	0.0004	0.0001	0.0006	0.0001	0.0004	0.0004	0.0003	0.0004	0.0005	0.0002	0.0006	0.0002	0.0006	0.0003	0.0003	0.0010	0.0008	0.0003	0.0005	0.0000
17	Weight changes effect	(66)	-0.0043	0.0119	-0.0093	-0.0107	-0.0038	-0.0086	-0.0042	0.0050	0.0044	0.0021	0.0048	0.0016	-0.0046	0.0022	0.0121	-0.0029	0.0025	-0.0031	0.0102	-0.0198	-0.0010	0.0089	0.0024	0.0135	0.0003	-0.0090
18	Self generated risk premium	(63)	-0.0042	0.0176	-0.0087	-0.0149	-0.0041	-0.0071	-0.0032	0.0058	0.0054	0.0029	0.0061	0.0024	-0.0065	0.0008	0.0146	-0.0019	0.0057	-0.0031	0.0099	-0.0243	-0.0016	0.0129	0.0028	0.0127	0.0035	-0.0090
	Optimal weight composition																											
19	Alpha		0.41%	26.68%	-1.59%	-10.61%	-3.37%	7.73%	2.46%	4.63%	1.65%	3.31%	9.77%	2.06%	-15.62%	-18.46%	18.39%	6.13%	5.69%	-1.19%	-6.23%	-34.90%	-9.24%	10.10%	-2.29%	-6.39%	25.45%	1.57%
20	Crossed beta		-0.04%	0.60%	3.33%	1.11%	1.48%	1.87%	1.87%	0.64%	3.25%	0.52%	4.53%	2.36%	2.36%	4.31%	4.77%	1.41%	1.43%	1.04%	4.27%	2.42%	3.41%	3.40%	5.11%	1.76%	5.00%	-0.57%
21	Weight changes		-14.67%	57.31%	-26.71%	-24.11%	-34.19%	-54.14%	-18.59%	34.58%	22.88%	10.80%	52.63%	8.67%	-32.22%	22.83%	109.60%	-22.46%	5.73%	-17.13%	67.88%	-143.28%	-9.70%	29.51%	16.16%	75.08%	3.05%	-105.30%
22	Explained weight		-14.30%	84.59%	-24.96%	-33.61%	-36.08%	-44.54%	-14.26%	39.85%	27.78%	14.63%	66.93%	13.09%	-45.48%	8.69%	132.77%	-14.92%	12.85%	-17.28%	65.93%	-175.76%	-15.53%	43.00%	18.98%	70.45%	33.49%	-104.31%
23	Unexplained weight		-0.06%	0.32%	0.11%	0.20%	0.39%	1.02%	0.16%	0.80%	0.05%	-0.01%	1.17%	0.43%	0.60%	1.36%	0.15%	-0.41%	-0.89%	-0.27%	0.60%	-0.42%	1.23%	0.15%	0.51%	0.00%	0.98%	-0.19%
24	Addition (optimal weight)		-14.35%	84.91%	-24.85%	-33.41%	-35.69%	-43.52%	-14.10%	40.65%	27.84%	14.62%	68.09%	13.52%	-44.88%	10.04%	132.91%	-15.32%	11.97%	-17.55%	66.54%	-176.18%	-14.30%	43.15%	19.49%	70.45%	34.47%	-104.49%
		Gini portfolio risk premium			0.0241		Gini portfolio standard deviation			0.0259																		

Figure 1: MEAN VARIANCE AND GINI PORTFOLIO PLOTS

MEAN VARIANCE



GINI